BOOK II

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CHAPTER I

PROGRESS OF ASTRONOMY

308. THE science of astronomy was cultivated very early, and many important observations and discoveries were made, yet no accurate inferences leading to the true system of the world were drawn from them, until a much later period. It is not surprising, that men deceived by appearances, occasioned by the rotation of the earth, should have been slow to believe the diurnal motion of the heavens to be an illusion; but the absurd consequence which the contrary hypothesis involves, convinced minds of a higher order, that the apparent could not be the true system of nature.

Many of the ancients were aware of the double motion of the earth; a system which Copernicus[1] adopted, and confirmed by the comparison of a series of observations, that had been accumulating for ages; from these he inferred that the precession of the equinoxes might be attributed to a motion of the earth's axis. He ascertained the revolution of the planets round the sun, and determined the dimensions of their orbits, till then unknown. Although he proved these truths by evidence which had ultimately dissipated the erroneous theories resulting from the illusions of the senses, and overcame the objections which were opposed to them by ignorance of the laws of mechanics, this great philosopher, constrained by the prejudices of the times, only dared to publish the truths he had discovered, under the less objectionable name of hypothesis.

In the seventeenth century, Galileo,[2] assisted by the discovery of the telescope, was the first who saw the magnificent system of Jupiter's satellites, which furnished a new analogy between the planets and the earth: he discovered the phases of Venus, by which he removed all doubts of the revolution of that planet round the sun. The bright spots which he saw in the moon beyond the line which separates the enlightened from the obscure part, showed the existence and height of its mountains. He observed the spots and rotation of the sun, and the singular appearances exhibited by the rings of Saturn; by which discoveries the rotation of the earth was confirmed: but if the rapid progress of mathematical science had not concurred to establish this essential truth, it would have been overwhelmed and stifled by fanatical zeal. The opinions of Galileo were denounced as heretical by the Inquisition, and he was ordered by the Church of Rome to retract them. At a late period he ventured to promulgate his discoveries, but in a different form, vindicating the system of Copernicus; but such was the force of superstition and prejudice, that he, who was alike an honour to his country, and to the human race, was again subjected to the mortification of being obliged to disavow what his transcendent genius had proved to be true. He died at Arcetri[3] in the year 1642, the year in which Newton[4] was born, carrying with him, says Laplace,[5] the regret of Europe, enlightened by his labours, and indignant at the judgment pronounced against him by an odious tribunal.

The truths discovered by Galileo could not fail to mortify the vanity of those who saw the earth, which they conceived to be the centre and primary object of creation, reduced to the rate of but a small planet in a system, which, however vast it may seem, forms but a point in the scale of the universe.

The force of reason by degrees made its way, and persecution ceased to be the consequence of stating physical truths, though many difficulties remained to impede its progress, and no ordinary share of moral courage was required to declare it: '*prejudice,*' says an eminent author, '*bars up the gate of knowledge; but he who would learn, must despise the timidity that shrinks from wisdom, he must hate the tyranny of opinion that condemns its pursuit: wisdom is only to be obtained by the bold; prejudices must first be overcome, we must learn to scorn names, defy idle fears, and use the powers of nature to give us the mastery of nature. There are virtues in plants, in metals, even in woods, that to seek alarms the feeble, but to possess constitutes the mighty.*

About the end of the sixteenth, or the beginning of the seventeenth century, Tycho Brahe[6] made a series of correct and numerous observations on the motion of the planets, which laid the foundation of the laws discovered by his pupil and assistant, Kepler.[7]

Tycho Brahe, however, would not admit of the motion of the earth, because he could not conceive how a body detached from it could follow its motion: he was convinced that the earth was at rest, because a heavy body, falling from a great height, falls nearly at the foot of the vertical.

Kepler, one of those extraordinary men, who appear from time to time, to bring to light the great laws of nature, adopted sounder views. A lively imagination, which disposed him eagerly to search for first causes, tempered by a severity of judgment that made him dread being deceived, formed a character peculiarly fitted to investigate the unknown regions of science, and conducted him to the discovery of three of the most important laws in astronomy.

He directed his attention to the motions of Mars, whose orbit is one of the most eccentric in the planetary system, and as it approaches very near the earth in its oppositions, the inequalities of its motions are considerable; circumstances peculiarly favorable for the determination of their laws.

He found the orbit of Mars to be an ellipse, having the sun in one of its foci; and that the motion of the planet is such, that the radius vector drawn from its centre to the centre of the sun, describes equal areas in equal times. He extended these results to all the planets, and in the year 1626, published the Rudolphine Tables,[8] memorable in the annals of astronomy, from being the first that were formed on the true laws of nature.

Kepler imagined that something corresponding to certain mysterious analogies, supposed by the Pythagoreans[9] to exist in the laws of nature, might also be discovered between the mean distances of the planets, and their revolutions around the sun: after sixteen years spent in unavailing attempts, he at length found that the squares of the times of their sidereal revolutions are proportional to the cubes of the greater axes of their orbits; a very important law, which was afterwards found equally applicable to all the systems of the satellites. It was obvious to the comprehensive mind of Kepler, that motions so regular could only arise from some universal principle pervading the whole system. In his work *De Stella Martis,*[10] he observes, that[11] '*two insulated bodies would move towards one another like two magnets, describing spaces reciprocally as their masses. If the earth and moon were not held at the distance that separates them by some force, they would come in contact, the moon describing* $\frac{53}{54}$ *of the distance, and the earth the remainder, supposing them to be equally dense.*' '*If,*' he continues, '*the earth ceased to attract the waters of the ocean, they would go to the moon by the attractive force of that body. The attraction of the moon, which extends to the earth, is the cause of the ebb and flow of the sea.*' Thus Kepler's work, *De Stella Martis*, contains the first idea of a principle which Newton and his successors have fully developed.

The discoveries of Galileo on falling bodies, those of Huygens[12] on Evolutes,[13] and the centrifugal force, led to the theory of motion in curves. Kepler had determined the curves on which the planets move, and Hook[e][14] was aware that planetary motion is the result of a force of projection combined with the attractive force of the sun.

Such was the state of astronomy when Newton, by his grand and comprehensive views, combined the whole, and connected the most distant parts of the solar system by one universal principle.

Having observed that the force of gravitation on the summits of the highest mountains is nearly the same as on the surface of the earth, Newton inferred that its influence extended to the moon, and, combining with her force of projection, causes that satellite to describe an elliptical orbit round the earth. In order to verify this conjecture, it was necessary to know the law of the diminution of gravitation. Newton considered, that if terrestrial gravitation retained the moon in her orbit, the planets must be retained in theirs by their gravitation to the sun; and he proved this to be the case, by showing the areas to be proportional to the times: but it resulted from the constant ratio found by Kepler between the squares of the times of revolutions of the planets, and the cubes of the greater axes of their orbits, that their centrifugal force, and consequently their tendency to the sun, diminishes in the ratio of the squares of their distances from his centre. Thus the law of diminution was proved with regard to the planets, which led Newton to conjecture, that the same law of diminution takes place on terrestrial gravitation.

He extended the laws deduced by Galileo from his experiments on bodies falling at the surface of the earth, to the moon; and on these principles determined the space she would move through in a second of time, in her descent towards the earth, if acted upon by the earth's attraction alone. He had the satisfaction to find that that the action of the earth on the moon is inversely as the square of the distance, thus proving the force which causes a stone to fall at the earth's surface, to be identical with that retains the moon in her orbit.

Kepler having established the point that the planets move in ellipses, having the sun in one of their foci, Newton completed his theory, by showing that a projectile might move in any of the conic sections, if acted on by a force directed to the focus, and inversely as the square of the distance: he determined the conditions requisite to make the trajectory a circle, an ellipse, a parabola, or hyperbola. Hence he also concluded, that comets move round the sun by the same laws as the planets.

A comparison of the magnitude of the orbits of the satellites and the periods of their revolutions, with the same quantities relatively to the planets, made known to him the respective masses and densities of the sun and of planets accompanied by satellites, and the intensity of gravitation at their surfaces. He observed, that the satellites move round their planets nearly as they would have done, had the planets been at rest, whence he concluded that all these bodies obey the same law of gravitation towards the sun: he also concluded, from the equality of action and re-action, that the sun gravitates towards the planets, and the planets towards their satellites; and that the earth is attracted by all bodies which gravitate towards it. He afterwards extended this law to all the particles of matter, thus establishing the general principle, that each particle of matter attracts all other particles directly as its mass, and inversely as the square of its distance.

These splendid discoveries were published by Newton in his *Principia*,[15] a work which has been the admiration of mankind, and which will continue to be so while science is cultivated.

Referring to that stupendous effort of human genius, Laplace, who perhaps only yields to Newton in priority of time, thus expresses himself in a letter to the writer of these pages:[16]

'*Je publie successivement les divers livres du cinquième volume qui doit terminer mon traité de* ***Mécanique Céleste****, et dans lequel je donne l'analyse historique des recherches des géomètres sur cette matière. Cela m'a fait relire avec une attention particulière l'ouvrage incomparable des* ***Principes Mathématiques*** *de la philosophie naturelle de Newton, qui contient le germe de toutes ces recherches. Plus j'ai étudié cet ouvrage, plus il m'a paru admirable, en me transportant surtout à l'époque où il a été publié. Mais en même tems que j'ai senti l'élégance de la méthode synthétique suivant laquelle Newton a présenté ses découvertes, j'ai reconnu l'indispensable nécessité de l'analyse pour approfondir les questions très difficiles qu'il n'a pu qu'effleurer par la synthèse. Je vois avec un grand plaisir vos mathématiciens se livrer maintenant à l'analyse; et je ne doute point qu'en suivant cette méthode avec la sagacité propre à votre nation, ils ne soient conduits à d'importantes découvertes*.'

The reciprocal gravitation of the bodies of the solar system is a cause of great irregularities in their motions; many of which had been explained before the time of Laplace, but some of the most important had not been accounted for, and many were not even known to exist. The author of the *Mécanique Céleste* therefore undertook the arduous task of forming a complete system of physical astronomy, in which the various motions in nature should be deduced from the first principles of mechanics. It would have been impossible to accomplish this, had not the improvements in analysis kept pace with the rapid advance in astronomy, a pursuit in which many have acquired immortal fame; that Laplace is pre-eminent amongst these, will be most readily acknowledged by those who are best acquainted with his works.

Having endeavoured in the first book to explain the laws by which force acts upon matter, we shall now compare those laws with the actual motions of the heavenly bodies, in order to arrive by analytical reasoning, entirely independent of hypothesis, at the principle of that force which animates the solar system. The laws of mechanics may be traced with greater precision in celestial space than on earth, where the results are so complicated, that it is difficult to unravel, and still more so to subject them to calculation: whereas the bodies of the solar system, separated by vast distances, and acted upon by a force, the effects of which may be readily estimated, are only disturbed in their respective movements by such small forces, that the general equations comprehend all the changes which ages have produced, or may hereafter produce in the system; and in explaining the phenomena it is not necessary to have recourse to vague or imaginary causes, for the law of universal gravitation may be reduced to calculation, the results of which, confirmed by actual observation, afford the most substantial proof of its existence.

It will be seen that this great law of nature represents all the phenomena of the heavens, even to the most minute details; that there is not one of the inequalities which it does not account for; and that it has even anticipated observation, by unfolding the causes of several singular motions, suspected by astronomers, but so complicated in their nature, and so long in their periods, that observation alone could not have determined them but in many ages.

By the law of gravitation, therefore, astronomy is now become a great problem of mechanics, for the solution of which, the figure and masses of the planets, their places, and velocities at any given time, are the only data which observation is required to furnish. We proceed to give such an account of the solution of this problem, as the nature of the subject and the limits of this work admit of.

Notes

[1] Copernicus, Nicolas, 1473-1543, astronomer and founder of the heliocentric world system, born in Torun, Poland. The heliocentric system was advanced in his *De revolutionibus orbium coelestium* (1543). The view eliminated several problems in Ptolemy's (see note 15, *Preliminary Dissertation*) geostatic model, but was no more accurate in predicting celestial motions and really no simpler. Copernicus, for example, used 44 more epicycles that Ptolemy. Furthermore, the main counter-argument against the Copernican system, the lack of observed stellar parallax, was not adequately explained; nor was there an effective defense of the new model in terms of Aristotle's physics on which Copernicus still relied. In addition his work relied on scant observational evidence. The work nonetheless became the cornerstone for modern astronomical science.

[2] See note 1, *Introduction.*

[3] Arcetri is the small village near Florence where Galileo remained under house arrest until his death.

[4] Galileo died on Jan 4, 1642. Newton was bom on Christmas day of the same year.

[5] Somerville uses the spelling *La Place* throughout the text.

[6] Brahe, Tycho, 1546-1601 astronomer, born in Knudstrup, Sweden. Brahe was known for his unprecedented observational accuracy. He operated an observatory called Uraniborg on the island of Ven where most of his observations were conducted. He rejected the Copernican theory (see note 1) and proposed an independent geocentric system of the world (the Tyconic system) which is mathematically equivalent to the Copernican system. In his system the earth remains stationary, but the five remaining planets revolve about the sun, which in tun revolves about the earth. After his death Kepler used Brahe's precise data to demonstrate the elliptical orbit of Mars and to establish his three laws of planetary motion which later formed the foundation for Newtonian mechanics.

[7] See note 3, *Preliminary Dissertation.*

[8] The Rudolphine Tables were the first to make use of Kepler's newly formulated Laws on planetary motions, calibrated using Tycho Brahe's (see note 6) store of accurate planetary observations. They received a spectacular validation on November 7, 1631, when the French philosopher and sometimes astronomer Pierre Gassendi (1592-1655) observed a transit of Mercury across the solar disk, as predicted by Kepler. Kepler's prediction of this event was far more accurate than those based on the Copernican Tables. This success paved the way for the general acceptance not only of the Rudolphine Tables, but also by extension, of Kepler's three Laws of planetary motions. *Paul Charbonneau, High Altitude Observatory (NCAR)*.

[9] See note 10.

[10] Kepler, Johannes, 1571-1630, *Astronomia nova aitiologetos, seu physica coelestis, tradita commentariis de motibus stellae Martis, ex observationibus G.V. Tychonis Brahe ... elaborata ...*, Pragae, 1609. (see also note 2, *Preliminary Dissertation.*)

[11] Not italicized in the 1st edition.

[12] See note 12, *Book I, Chapter II* .

[13] *Evolutes*. The locus of the centers of curvature of a given curve, *The American Heritage® Dictionary*, *1996.*

[14] Robert Hooke (1635-1703), chemist and physicist, born in Freshwater, Isle of Wight, England. Hooke formulated the law governing elasticity (Hooke's law), and invented the balance spring for watches. His most important work is his *Micrographia* (1665).

[15] Newton, Isaac, Sir, 1642-1727, *Isaac Newton's Philosophiae naturalis principia mathematica*, Cambridge: Cambridge University Press, 1972. (see also note 1, *Preliminary Dissertation.*)

[16] "I am publishing consecutively the various books of the fifth volume which will complete my treatise of Celestial Mechanics, and in which I give an historical analysis of the investigations of the geometers into this matter. This made me reread with particular attention the incomparable work of Newton, *The Mathematical Principles of Natural Philosophy* (1687), which contains the germ of all this research. The more I studied this work, the more it appeared to me admirable, transporting me, above all, to the period when it was published. But at the same time as I sensed the elegance of the synthetic method according to which Newton presented his discoveries, I recognized the indispensable necessity of analysis for plumbing the depths of the very difficult questions that one could only treat superficially by synthesis. I note with great pleasure that your mathematicians now devote themselves to analysis; and I have no doubt whatsoever that by following this method with the sagacity characteristic of your nation, they will be led to important discoveries." *Translation by John Black, Malaspina University-College.*