BOOK III

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CHAPTER V

EFFECTS OF THE SECULAR VARIATION IN THE PLANE OF THE ECLIPTIC

780. HAVING developed all the inequalities to which the moon is subject, we shall now show that the secular variation in the plane of the ecliptic has no effect on the inclination of the lunar orbit.

The latitude of the earth s', being extremely small, was omitted in the values of *R*, No. (208): it can only arise from disturbances either secular or periodic: both oscillate between fixed limits; but we shall suppose s' to relate only to the secular variations in the plane of the ecliptic, and according to equations (138) shall only assume it to be equal to a series of terms of and form,

$$\sum K\cdot\sin\left(v'+it+\in\right),$$

i being a very small coefficient. Then omitting quantities of the order s^3, the tangent of the moon's latitude is

$$s=g\;\sin\left(gv-q\right)+\sum K\sin\left(v+it+\in\right)+d\,s\,;$$

equation (205), which determines the latitude, is

$$0=\frac{d^2s}{dv^2}+s+\frac{3m'u'^3s}{2h^2u^4}-\frac{3mu'^3s}{2h^2u^4}-\frac{3mu'^3s}{h^2u^4}\cos\left(v-v'\right)+\frac{3mu'^3s}{2h^2u^4}d\,s\,.$$

Now

$$\frac{3m'u'^3s}{2h^2u^4}=\frac{3m^2}{2}\cdot\frac{a'}{a}\sum K\sin\left(v+it+\in\right).$$

The following term gives the same quantity with a contrary sign. And if

$$d\,s=\sum bK\sin\left(v+it+\in\right),$$

the last term gives

$$\frac{3m^2}{2}\frac{a'}{a}\sum bK\sin\left(v+it+\in\right),$$

so that the differential equation of the moon's latitude becomes

$$0=\frac{d^2s}{dv^2}+s+\frac{3m^2}{2}\cdot\frac{a'}{a}\sum bK\sin\left(v+it+\in\right)$$

and if $\sum bK\sin(v+it+\in)$ be put for $\frac{d^2s}{dv^2}+s,$ the equation becomes

$$0=\sum(1+b)K\left\{1+(1+i)^2\right\}\sin(v+iv+\in)+\frac{3m^2}{2}\frac{a'}{a}\sum bK\sin(v+iv+\in)$$

for *iv* may be put for it, whence

$$b=-\frac{1-(1+i)^2}{1-(1+i)^2+\frac{3m^2}{2}\cdot\frac{a'}{a}}=\frac{2i+i^2}{\frac{3m^2}{2}\cdot\frac{a'}{a}-2i-i^2}.$$

Hence the variation of *s*, the moon's latitude, with regard to the secular motion of the ecliptic is [1]

$$\frac{\sum\left(2i+i^2\right)K\sin(v+iv+\in)}{\frac{3m^2}{2}\cdot\frac{a'}{a}-2i-i^2}$$

This quantity is insensible, for *iv* is only about $16''$ a year, and

$$\frac{3m^2}{3}\cdot\frac{a'}{a}$$

being nearly $40^\circ\ 37'$, the value of the factor

$$\frac{2i+i^2}{\frac{3m^2}{2}\cdot\frac{a'}{a}-2i-i^2}$$

is only $0''.00022$.

So that the ecliptic in its motion carries the orbit of the moon along with it.

787. The coincidence of theory with observation, in explaining the inequalities in the motions of the moon, affords the most conclusive proof of the universality of the law of gravitation. Having deduced all these inequalities from that one cause, Laplace[2] established the correctness of the results obtained by analysis by comparing them with the lunar tables computed by Mason[3] from 1137 observations made by Bradley[4] between the years 1750 and 1760, and corrected by Burg[5] by means of upwards of 3000 observations made by Maskelyne[6] between the years 1765 and 1793. He had the satisfaction to find that the greatest difference did not exceed $8''$ in the longitude, while the difference in latitude was only $1''.94$, a degree of accuracy sufficient to warrant the tables of latitude being regarded as equivalent to the result of theory: the approximations in latitude, indeed, are more simple and convergent than those in longitude. The inequalities in the lunar parallax are so small, that theory will determine them more correctly than observation. Accurate as these results are, it is still possible that the motions of the moon may be affected by the resistance of an ethereal medium surrounding the sun.

Notes

[1] The term i^2 in the denominator *reads* $i2$ in the 1st edition.

[2] See note 4, *Introduction*.

[3] Mason, Charles, 1730-1787, astronomer, known for the "Mason–Dixon Line" in the USA. He was an assistant at Greenwich Observatory, and observed the 1761 transit of Venus at the Cape of Good Hope with the English surveyor Jeremiah Dixon.

[4] See note 38, *Preliminary Dissertation*.

[5] See note 3, *Bk. III, Chap. II.*

[6] See note 55, *Bk. II, Chap. VI.*