

CRITICAL REVIEWS OF *MECHANISM OF THE HEAVENS*

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THE LITERARY GAZETTE , AND JOURNAL OF THE BELLES LETTRES

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WE opened this book with no inconsiderable apprehensions for the reputation—we mean the scientific and literary reputation—of the fair author; for although Mrs. Somerville has long been considered, by persons acquainted with such subjects, as one of the most accomplished and most highly informed mathematicians of the day, no public evidence, antecedent to the appearance of this work, has been afforded of the correctness of this very high praise. We felt, therefore, the deepest interest in the result of the gigantic experiment our countrywoman undertook to perform, which was no less than to give to the world a succinct, profound, but, at the same time, as popular a view as possible of the great Laplace's *Mécanique Céleste*.¹

A mere translation of that work would of itself have been a formidable task; and we may remark, by the way, that the distinguished American mathematician, Bowditch,² has already given to the world a portion of a translation, illustrated by copious notes, which cannot fail to be of the highest value to the student in those intricate pursuits. Mrs. Somerville's object, however, was of a different order, and one more consonant to the boldness and vigour of an original thinker, conscious of adequate powers to invest even the most abstruse topic with the virgin interest which true genius alone can create. She saw that Laplace's book was sealed to all who were not in familiar possession of that marvelous language in which his history of the heavens is exclusively written, and without which familiar acquaintance, the study became one of almost hopeless labour. Mrs. Somerville had fortunately obtained not only all the requisite knowledge to understand Laplace's exposition of the subject, but believed that she could facilitate its acquisition by others; and she conceived the energetic and public-spirited idea of acting as interpreter between the great continental successor of Newton and the less-instructed mathematicians, astronomers, and, we may add, general readers of her native country.

From the dedication, which we have much pleasure in stating is addressed, with singular taste and propriety, to Lord Brougham,³ it appears that we are indebted to the sagacity of this extraordinary person for having first suggested the undertaking, and to his great influence afterwards in securing its accomplishment. His lordship, it seems, wished to embody Laplace's work in the publications of the Society for the Diffusion of Useful Knowledge; but, as may readily be supposed, Mrs. Somerville could not accomplish this purpose. Now, however, that the first grand step has been made towards giving a popular character to the highest flights of astronomical knowledge, we believe the object which the lord chancellor had in view no longer impossible, but only under one condition of the problem, namely, that the master-hand which originally gave the impulse shall undertake its completion.

It would be quite foreign to the purposes and habits of our Journal to give even the slightest sketch of the work before us; in fact, as the author says in her introduction, "To

¹ See note 4, *Introduction*.

² See note 3, *Foreword to the Second Edition*.

³ See note 35, *Foreword to the Second Edition*.

accomplish the task of giving an account of the '*Mécanique Céleste*' without having recourse to the higher branches of mathematics is impossible;" and as we cannot pretend to have time (although, of course, we have the knowledge) to do the topic justice, we shall spare our readers the shock of a whole army of figures and symbols, with which we might cover our pages, if we were disposed to shew off our learning. As we despise such ostentation, for reasons best known to ourselves, we shall rest content with alluding to the most obvious distinction between this work and its great original, and then advert to the strictly popular branch of the undertaking.

"Diagrams," says Mrs. Somerville, "are not used in Laplace's works, being unnecessary to those versed in analysis: some, however, will be occasionally introduced for the convenience of the reader."

We do not know what meaning others may attach to the word occasionally, but to us it gave no idea of the extent of the assistance which this departure from the plan of Laplace is calculated to afford. Of course, this is not done throughout; for although, as our author says, "many subjects admit of geometrical demonstration, yet the object of the work being rather to give the spirit of Laplace's method, than to pursue a regular system of demonstration, it would be a deviation from the unity of his plan to adopt it in this."

For all this, we have good authority for saying, that the student who really wished to understand the mechanism of the heavens as developed in that splendid work, will here discover, by this and other aids furnished by Mrs. Somerville, the readiest means that are possible of acquiring that knowledge. Our author well characterizes Laplace's work as the resolution of a "a great problem in dynamics, wherein it is required to deduce all the phenomena of the solar system from the abstract laws of motion, and to confirm the truth of those laws by comparing theory with observation."

Still, however, had Mrs. Somerville confined herself to the mighty task just described, and which she has executed with a degree of address every way worthy of her principal, she would have fallen short of our wishes. We should have felt greatly disappointed, indeed, if she had not condescended to give the host of general readers some conception of the wonders concealed from the sight under the mystic garb of the differential calculus. With great good sense, therefore, and no small kindness, Mrs. Somerville has given all that we could have desired, in a preliminary dissertation, which, independently of its own intrinsic excellence, cannot fail to stimulate many readers to pursue for themselves the investigation of the phenomena it describes.

We possess already innumerable discourses on astronomy, in which the wonders of the heavens and their laws are treated of; but we can say most conscientiously, that we are acquainted with none—not even Laplace's own beautiful *exposé* in his *Système du Monde*—in which all that is essentially interesting in the motions and laws of the celestial bodies, or which is capable of popular enunciation, is so admirably, so graphically, or we may add, so unaffectedly and simply placed before us. The style is luminous and precise throughout, totally without ambition, either in thought or expression, and untouched by any depreciating apologies as to the execution, or marred by any feebleness in the design. We see nothing of the author, and think only of the subject; and it is quite clear, from the strongest possible internal evidence, that while she was penning this dissertation, a single thought of self never once crossed her mind. She felt—she must have felt—perfectly competent to treat the subject as it ought to be treated; and under this conviction, gave most spontaneous currency to her knowledge. That such perfection in style is the result of labour, at some time or other, we hold to be quite as certain as any proposition demonstrated in this book; but we are also quite sure, that the ease, vigour, and

clearness, throughout such a dissertation as this, can spring only from the completest familiarity with the subject in all its bearings, chastened by the single-hearted purpose of telling what is to be told in the plainest and most acceptable language.

Is it asking too much of Mrs. Somerville to express a hope that she will allow this beautiful preliminary dissertation to be printed separately, for the delight and instruction of thousands of readers, young and old, who cannot understand, or who are too indolent to apply themselves to the elaborate parts of the work? If she will do this, we hereby promise to exert our best endeavours to make its merits known. As present we have left ourselves no space to enter into the analysis we shall be delighted to be again called upon to undertake; and we can only repeat, that we are not acquainted with any account of the celestial movements which is at once so complete in all its parts, and yet so judiciously condensed. Indeed, when we came to the conclusion, we felt only regret that our intellectual feast was so short: but on reading it again, we discovered much more matter for careful reflection than we had discovered when hurried along by the witchery of the style, or seduced into new curiosity by the evergreen freshness of this delicious subject. For of astronomy it may be more truly said, that almost of any other science, that the further we advance, the greater is our desire to proceed. In this pursuit every thing is pure, serene, certain. It is truly the “image of eternity, the throne of the invisible,” that we are then contemplating; and the mind which is not raised by such contemplation above the selfish object and angry passions of this earth, must be gross indeed. But we must not forget that it involves still higher and more important considerations, by teaching us at once the wisdom, the power, and the beneficence of God, the Creator of all these things. And it must go hard indeed with our hearts if they be not touched by these important proofs of the Divine goodness to the creatures he has placed on one of the smallest of the countless myriads of orbs he has set in motion.

The following passage, with which we shall conclude this notice, is a good specimen of our fair author’s style and is much in point.⁴

“The heavens afford the most sublime subject of study which can be derived from science: the magnitude and splendour of the objects, the inconceivable rapidity with which they move, and the enormous distances between them, impress the mind with some notion of the energy that maintains them in their motions with a durability to which we can see no limits. Equally conspicuous is the goodness of the great First Cause in having endowed man with faculties by which he can not only appreciate the magnificence of his works, but trace, with precision, the operation of his laws, use the globe he inhabits as a base wherewith to measure the magnitude and distance of the sun and planets, and make the diameter of the earth’s orbit the first step of a scale by which he may ascend to the starry firmament. Such pursuits, while they ennoble the mind, at the same time inculcate humility, by showing that there is a barrier, which no energy, mental or physical, can ever enable us to pass: that however profoundly we may penetrate the depths of space, there still remain innumerable systems, compared with which those which seem so mighty to us must dwindle into insignificance, or even become invisible; and that not only man, but the globe he inhabits, nay the whole system of which it forms so small a part, might be annihilated, and its extinction be unperceived in the immensity of creation.”

⁴ See *Preliminary Dissertation*.

THE ATHENAEUM

[Charles Buller?]⁵

221 (1832), p. 43-44, January, 1832

WE have universities, a considerable portion of whose vast revenues is annually paid for the support of *men* of science, and a further portion annually set apart for the printing of *books* on science. How is it that no English edition of the *Mécanique Céleste* has hitherto appeared under the sanction of a learned body and a respectable editor? If other evidence of the decline of science in this country were wanting, a strong case of suspicion might be grounded upon this one fact.⁶

It is recorded on the authority of the *Edinburgh Review*, that some fifteen years ago the British empire did not contain six individuals sufficiently learned in the exact sciences, to read this work; and here we have, at the hands of a lady, the very spirit and essence of its four quarto volumes and supplements, in a single octavo. In the preface to her book, Mrs. Somerville very properly gives us some account of its parentage. Lord Brougham,⁷ it appears, was father to the thought,—having expressed a wish that its talented authoress would endeavour to introduce the working classes to a knowledge of the doctrines of the *Mechanique*—a wish which, conceived in the very spirit of that boundless philanthropy for which his Lordship is remarkable, and encouraged by the Society for the Diffusing [sic] of Useful Knowledge, is realized in the work before us.

We are convinced that the gratitude of the working classes would be unlimited, could they but appreciate the extent of the obligation. We are not, however, sanguine on this subject. With the very best wished for the general diffusion of knowledge, we do not expect, for many years, to find the work of Laplace much read among the labouring poor; and, indeed, looking at the splendour of the typography of the volume before us, and the patrician name of the bibliopole, we are disposed to think that Mrs. Somerville herself never seriously contemplated an early period

Contrectatus ubi manibus sordescere vulgi coeperit:

⁵ Buller, Charles, (1806-1848) an M.P. for West Looe in Cornwall and a frequent contributor to the *Athenaeum*. Buller had earlier attacked Somerville's book in the House of Commons. Although the style used in this review reflects Buller's proclivity for "ready taunts and a prankish disposition," the author is not identified in the review article. Elizabeth Patterson characterizes the review as "mockingly and patronizingly derisive" of both Somerville and in its accompanying "ridicule of Brougham and the Society for the Diffusion of Useful Knowledge for their illusion that the working class wished to be introduced to Laplace and for their foolishness in choosing a woman to undertake such a task." Patterson goes on to note that the "words and tone demonstrate disdain for learned women, while its mathematical contents reveal its writer's inadequacies in that discipline." In her autobiography Somerville recalls how "a Mr. Buller member for someplace I have forgotten in the west of England spoke of...my book with sovereign contempt. I was much annoyed more so than I ought to have been for he showed that he was totally ignorant of the state of science." (Patterson, Elizabeth Chambers, *Mary Somerville and the Cultivation of Science*, International Archives of the History of Science, Martinus Nijhoff Pub., 1983, p. 83-4.)

⁶ An English translation of Laplace is at present publishing at Boston, in North America, one volume of which has found its way to this country. The translator is Mr. Bowditch. The text is excellently printed, and accompanied by notes. (note in *Athenaeum* Review.)

⁷ See note 35, *Foreword to the Second Edition*.

There is reserved for it a higher destiny than the hands of the unwashed. We behold it, in our critical imagination, reposing in graceful indolence on the table of every confirmed blue of the United Kingdom; its leaves will be cut; its pages turned over, by the fair hands of the very fairest of created things—and not more fair than wise. On the mysterious symbols which so mysteriously shadow forth its meaning, there will dwell (in beautiful wonder,) the brightest eyes that, since the days of our first mother, have shone, for evil or for good, upon the less fortunate portion of humanity. What a world of delightful prattle it will originate! And then, when the novelty of its youth has passed away, how dignified, how conspicuous a place will be assigned to it in the library!—how perfect, how uninterrupted will be its retirement! A more complete realization of the “*otium cum dignitate*” of a book cannot be imagined.

Although we have long considered an English translation of Laplace the great desideratum in our science, yet we confess that, when the rumour was brought to us that such a work had been undertaken by a lady, we found the information somewhat comfortless—all the chances appeared to us to be against her success. We foresaw, in the promised translation, an occasional echo of that understanding of his doctrines which had established itself in her own mind, and the prospect was discouraging. Our critical discomfort arrived, however, at a maximum, when, on opening the book, we found it blazoned in the preface that, instead of a translation, we had the spirit of Laplace, according to Mrs. Somerville, bottled up in an octavo. The gloomiest of our forebodings had never led us to dream that the sacrilege of remodeling the thoughts of Laplace would be otherwise than an occasional evil, insinuating itself, as it were, upon the task of the translator: we were utterly unprepared to find it thus openly avowed.

Laplace is perfectly competent to convey his meaning in his own words: his style is simple, and yet full of power; his words a fitting vehicle for the sublime truths which they convey; and his method strictly logical. He was far too great a man to deal in verbiage; and it is our religious belief—that any person capable of *understanding* (we use the word emphatically) the mechanism of the heavens at all, will understand it best with his own pages. We want his work as fresh from his intellect as it can be brought to us through the medium of a translation; and we like not the task which Mrs. Somerville has undertaken, of giving us his thoughts in language different from that which he thought best calculated to convey them. If her object was to simplify his reasonings, we cannot but applaud the intention; but we have every excuse for not having observed it, inasmuch as the work itself laughs all simplicity to scorn. The following instances of lucid explanation are from the first page: “*The activity of matter seems to be a law of the universe, as we know of no particle at rest.*”

Now this proposition is manifestly true, provided always, that if the particle were at rest, we should know it. But we do not know this;—as Mrs. Somerville proceeds immediately to inform us; for

“*were a body absolutely at rest, we could not prove it to be so, because there are no fixed points to which it could be referred.*”

The argument therefore stands thus: The activity of matter would seem to us to be a law of the universe, provided that, if any particle (of whose existence we are conscious) were at rest, we should know it, and that we know of no such particle at rest. But the particles of matter may be at rest, and we do not know it: therefore, the activity of matter does *not seem* to us to be a law of the universe.

This is the first proposition laid down in Mrs. Somerville’s book; it is particularly unfortunate. We continue the quotation:—

“Consequently, if only one particle of matter were in existence, it would be impossible to determine whether it were at rest or in motion.”

Now, we submit that the rest or unrest of this solitary particle of matter, would remain equally in doubt, were the world ever so thickly peopled with particles, provided there were no one point known to be at rest. Mrs. Somerville proceeds:—

“Thus, being totally ignorant of absolute motion, relative motion forms the subject of investigation: a body is therefore said to be in motion, when it changes its position, with regard to other bodies which are said to be at rest.”

We, for our own parts, protest against Mrs. Somerville’s comprehensive admission of ignorance. It seems to us pretty plain, that relative motion cannot exist without absolute motion. Now, of this relative motion we are allowed to know something; we are not therefore *totally* ignorant of absolute motion.

We have give the whole of the first sentence of *Mechanism of the Heavens*; we will now give that of the *Mécanique Céleste*.

“A body appears to us to move, when it changes its situation with reference to a system of bodies which we consider at rest; but, as all bodies, even those which appear to us to enjoy the most absolute repose, may be in motion, we imagine a space without limits, immovable, and penetrable to matter: it is to the parts of this space, real or imaginary, that we refer, in thought, the positions of bodies; and we conceive them in motion, when they occupy, successively different situations in space.”

Our readers will perceive that Mrs. Somerville has framed her definition of motion according to that idea of it which Laplace has mentioned only to discard. Now it is to the discussion of this motion, with reference to which Mrs. Somerville and her author are thus at variance, that the whole work is devoted. It appears to us, from a careful consideration of the question, that in this first remarkable sentence of her book, Mrs. Somerville has endeavoured the whole universe to be in a state of unrest; in which she has failed, the proof being, *as she has shown*, impossible. She has then proceeded to establish the incontrovertible proposition, that there is no one point in the universe known to be at rest. From which proposition, laid down with a naiveté such as few could bring to so grave a discussion, she infers, that, if there were but one particle of matter in the universe, we should not know whether it were at rest or in motion—a useful conclusion, which leads her to terminate the discussion of absolute motion, by an admission of absolute ignorance.

On the subject of force, Mrs. Somerville is singularly unintelligible. We are not quite sure whether she admits the existence of a principle passing by that name or not. She talks of force *exerted by matter*—of matter *acting* upon matter—and much more in the same strain. At length, however, her mind grasps a definition; it is this:—“analytically

$$F = \frac{dv}{dt},$$

WHICH IS ALL WE KNOW ABOUT IT,”⁸

Spirit of the working classes, here is a boon! How admirable is the arrangement of symbols which thus concisely develops to us all that may be known of force. This is in the very spirit of that compression, by which an OCTAVO volume of mathematics is brought into the

⁸ Emphasis in original review.

compass of a threepenny pamphlet, and, at the same time, simplified from the intellectual standard of the well read student in physics to the mind of a mechanic.

Having thus told us all that is known of force, Mrs. Somerville proceeds, in the most natural manner in the world, to tell us something more, and then this over again. She afterwards becomes quite diffuse on the subject, and that so plausibly, that had she not before defined *all* that was *known* of force, we should have believed that we were really adding to our knowledge of it. In the fundamental proposition of the parallelogram of forces, Mrs. Somerville has replaced the demonstration of Laplace by that of Poisson [sic]⁹ or Pontécoulant,¹⁰ but by an old method now generally admitted to be no proof at all, and to be found in Dr. Wood's *Mechanics*.

We open the book casually at page 14, and we learn that the centre of curvature is the intersection of two normals—that, “*it never varies in the circle and sphere, because the curvature is everywhere the same.*” Now, it appears to us, that the term curvature, having no other than a conventional signification, dependent upon the position of the centre of curvature, it is beginning at the wrong end to argue a permanency of that position in any case from an equality of the curvature. The opposite is the true order of induction.

We find in the next sentence, that r being the radius of curvature, “*it is evident, that though it may vary from one point to another, it is constant for any one point, where $dr = 0$.*” Now, that for the *same point* the radius of curvature is the same, and for different points, different, we need not have been told, but how these facts involve the inference that $dr = 0$, escapes us.

The calculus of variations is dispatched *in a page*. In the theory of areas, the beautiful demonstration of Laplace is replaced by the method of the *Principia*. There appears to be few pages of the book which do not offer matter for similar animadversion: the subject will not, however, we fear, be interesting to the generality of our readers; we will therefore stop here.

Before we satisfy our critical conscience by recording an impartial opinion on the merits of a book, about which more than an unusual share of nonsense will, we foresee, be talked, we may be allowed to state, that we have risen from the perusal of it with the conviction, that Mrs. Somerville is a person of very extraordinary talents, and that we are possessed with an admiration, all but unlimited, for that what we understand to be the extent and variety of her attainments. Having said thus much, we feel ourselves compelled to add, that, in our belief, the work before us has been rashly undertaken, and very imperfectly completed; and that, remarkable as Mrs. Somerville's powers undoubtedly are, she has here assigned to herself a task considerably beyond them.

⁹ Poisson, Siméon Dennis, see note 1, *Bk. I, Chap. VI*. Poisson actually praised Somerville's book and urged her to continue through Laplace's other books. (Patterson, Elizabeth Chambers, *Mary Somerville and the Cultivation of Science*, International Archives of the History of Science, Martinus Nijhoff Pub., 1983, p. 87.)

¹⁰ See note 3, *Bk. II, Chap. IV*

EDINBURGH REVIEW

[Thomas Galloway]¹¹

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THIS unquestionably is one of the most remarkable works that female intellect ever produced, in any age or country; and with respect to the present day, we hazard little in saying, that Mrs. Somerville is the only individual of her sex in the world who could have written it. The higher branches of the mathematics are not among the recognized objects of female accomplishment; and accordingly the education of women is so directed, that they have rarely the means afforded them of acquiring even the elements of scientific knowledge. Hence if, prompted by curiosity, or the consciousness of a capacity for such studies, they attempt to deviate into a path in which only a few men of exalted genius have been able to make great progress, they must possess no ordinary strength of purpose and powers of application, if they avoid being repulsed at the very entrance. But, notwithstanding the difficulties inseparable from the pursuit of abstract truth, and the obstacles interposed by fashion and prejudice to render the results of science inaccessible to females, examples occasionally occur of individuals of that sex raising themselves to the very highest eminence in mathematical learning; as if to prove that they are no less capable of excelling in those studies which require the patient exercise of profound thought, than they are of adorning the higher walks of literature. Our learned readers will call to mind the beautiful and unfortunate Hypatia,¹² the commentator of Apollonius and Diophantus, and president of the Alexandrian school, whose attainments in all the sciences of her age have been depicted in such glowing terms as to render her an object of admiration to posterity. A modern, and equally illustrious example, is afforded by Agnesi,¹³ who, to a profound knowledge of mathematics, added an almost miraculous acquaintance with literature and philosophy, and gave the world, in her *Analytical Institutions*, a treatise which does honour not only to her sex, but to her age and country. The *Principia* of Newton, we may add, was translated into French by the celebrated *Marquise du Châtelet*,¹⁴—who thereby contributed, perhaps in no unimportant degree, to promote the knowledge of the Newtonian philosophy on the continent. With these illustrious names, that of Mrs. Somerville, already known in the annals of science, must henceforth be associated, on account of her great proficiency in the most sublime and difficult applications of mathematical analysis, evinced by this compend of the *Mécanique Céleste* of Laplace;—a work which, after the ample justice that has already been done to it in this Journal, and the unanimous decision of all who are capable of appreciating its merit, it would be superfluous, perhaps presumptuous, to undertake to criticise or to praise.

¹¹ Galloway, Thomas, mathematics master at Sandhurst and a student of Somerville's mentor William Wallace (1768-1843). Like Somerville, Wallace was self taught. Wallace and Somerville maintained a mathematical correspondence by mail (Patterson, Elizabeth Chambers, *Mary Somerville and the Cultivation of Science*, International Archives of the History of Science, Martinus Nijhoff Pub., 1983 , p. 5.)

¹² Hypatia of Alexandria (355-415).

¹³ Agnesi, Maria Gaetana, (1718-1799).

¹⁴ Châtelet, Gabrielle-Émilie Le Tonnelier de Breteuil, Marquise du, (1706-1749), French mathematician and physicist and mistress of Voltaire.

The publication of the *Mécanique Céleste*, forms an important epoch in the history of Physical Astronomy. In the course of that century of brilliant discovery which had elapsed since the appearance of the *Principia*, the different branches of analysis had been assiduously cultivated, and successfully applied to the computation of the greater part of the celestial phenomena. Difficulty after difficulty had yielded to the successive efforts of the illustrious men who, with emulous rivalry, undertook to develop the theory of gravity, till the mechanism of the solar system was completely revealed, and the whole science of astronomy founded on a single law. In the *Mécanique Céleste*, which embodied the results of their united labours and discoveries, the long series of proofs which had been begun by Newton was completed. Every inequality of the planetary motions which the most refined observation had been able to detect, as well as numerous others too minute to be sensible to observation, was referred to its immediate cause, and subjected to rigorous computation. All the changes which can take place in the system were explained, and included in formulae, which represent not merely its present state, but its past and future condition, even to remote ages.

Such was the sublime picture exhibited in that extraordinary production; but into none of the productions of the human intellect does time bring greater ameliorations than onto those of the mathematician. Although the *Mécanique Céleste* must ever continue—what it was described by its author to be—a monument to the genius of the age in which it was composed, it is already in some respects behind the actual state of science. Embracing most of the principle questions connected with the constitution of the universe and the laws of matter, it has furnished themes for the speculations of all succeeding geometers; the investigations have been re-considered under every different point of view of which they were susceptible; and numerous and important simplifications have been made, which have superceded the original methods. In one respect, indeed, the analytical theory of the system of the world is susceptible of indefinite improvement. Many of the problems it presents are of so difficult a nature, that the most powerful analysis is unable to grasp the solutions in a finite expression; in such cases, recourse must be had to successive approximations, and however far these may be pushed, the solutions obtained in this manner necessarily fall short of absolute accuracy. In the finite integration of formulae that have hitherto been found intractable; in the investigation of series that converge more rapidly; in the reduction of difficulties to classes, and rendering the methods already known more simple and uniform, ample scope will always remain for the exercise of the most inventive talent. The future results of analysis cannot, indeed, have that imposing character which belongs to the discovery of a primordial law of the universe, or of those beautiful relations which ‘bind and perpetuate the revolutions of nature,’ but in reference to the simplification and more general diffusion of science, they may still be of very great importance. The analytical processes by which the more refined truths of astronomy are reached, are of so abstruse a nature, and so far removed from ordinary apprehension, that they who contribute to render them more easily understood, may justly claim to be regarded as benefactors of science.

The work of which we are about to give an account, was originally intended, as appears from the dedication, to form one of the series of treatises published under the superintendence of the Society for the Diffusion of Useful Knowledge; but by reason of the great variety and importance of the subjects that Physical Astronomy presents for discussion, it unavoidable exceeded the limits of the Society’s publications. The very eminent nobleman,¹⁵ however, at whose request it is stated to have been undertaken, and to whom it is dedicated, still thought that

¹⁵ Lord Chancellor Brougham.

in its present form it might tend to promote the views of the Society: and under this high sanction it has been given to the world.

Mrs. Somerville has not very distinctly intimated the precise object she had in view in the composition of this treatise; and we are at some loss to discover whether an original work was contemplated, or merely an abridgement of that of Laplace. The only information given respecting its nature and purpose is contained in a sentence in the *Introduction*, in which it is said that '*in the following pages it is not intended to limit the account of the Mécanique Céleste to a detail of results, but rather to endeavour to explain the methods by which these results are deduced from one general equation of the motion of matter.*' From this we may infer, that while the main object was to demonstrate the results of the *Mécanique Céleste*, it was not intended to adhere strictly to the analysis of Laplace, but that the investigations would be rendered more simple and perspicuous where they admitted of improvement, and advantage taken of the recent discoveries in analysis to render the processes more comprehensive and uniform. This at least appears to have been the plan on which the work has been executed. In many cases the demonstrations of Laplace are given without alteration; in others they have been partially changed; and in a few instances they have been entirely supplanted by others drawn from other sources. Near the commencement, the explanations are full; as the work advances, and the difficulties increase, they become more rare; and in some of the most important problems the analysis of Laplace is transcribed without any explanation whatever. This, however, could hardly be otherwise. Indeed, when we consider the extent and abstruse nature of the various subjects that come under consideration, it will readily appear, that to give a clear and satisfactory explanation of the analytical methods of Laplace, without employing his own expressions, and exhibiting his own formulae, would be a task of no ordinary difficulty. The language of the calculus is the most concise by which human thought can be expressed; and when employed by so great a master, it receives a form which can rarely be altered without injury. The general style of Laplace is also remarkable for its perspicacity and precision; so that there is no hope of giving his meaning in different words with greater exactness, or more briefly.

With reference to the amount and species of explanation that the *Mécanique Céleste* may be thought to stand in need of, no general rule can be laid down; as all depends on the mathematical acquirements of the reader, and the direction his studies may have taken. Mrs. Somerville has evidently wished to render the theories of physical astronomy more accessible to those who have made only a moderate proficiency in analysis; but we fear, that in order to comprehend fully the *Mechanism of the Heavens*, little, very little, abatement can be made from the amount of mathematical knowledge which is indispensably required to enter with advantage or profit on the study of Laplace. In conformity with the practice of English writers, diagrams have been inserted '*for the convenience of the reader;*' and the analysis has been broken up into distinct propositions, by which means, without interrupting the process of investigation, the particular subject under discussion is set more prominently before the eye of the student. Such alterations, however, refer merely to the mode of representing the demonstrations, and do not at all touch the real difficulties. By some readers they will be probably be regarded as an impediment; and, in a mathematical investigation, it is obvious that whatever is not absolutely required to complete the chain of evidence, serves only to fatigue and distract attention. It may also be remarked, that the assistance which can be derived from the introduction of elementary illustrations into the higher problems of analysis, can only be partial and limited. From the first axioms of geometry to the sublime results of physical astronomy, the distance is immense; and if it were necessary to demonstrate every intermediate step, the bulk of a treatise containing these

results, would exceed all reasonable limits. However numerous the explanations may be, they can never supercede the necessity of a very extensive acquaintance with the abstract theories of pure mathematics; nor will it be found possible, by any amount of explanation whatever, to convert the *Mécanique Céleste* into an elementary treatise of dynamics.

The entire assemblage of methods and researches comprehended in the *Mécanique Céleste*, may be divided into three principle classes. The first relates to the translation of the bodies of the solar system in space, or the motions of their centres of gravity, supposing their masses to be united at those points. The second embraces the theory of their figures, their motions of rotation, and the positions and oscillations of their axes. The third is devoted to the consideration of a number of particular phenomena, including the oscillations of the fluids at the surfaces of planets, or rather of the earth; the aberration and refraction of light, and molecular attraction. Mrs. Somerville's work extends only to the first of these classes, and does not even include the comets. The subjects which come under discussion are consequently the trajectories described by each of the planets about the sun, and of the satellites about their primary planets; the forms, positions, and magnitudes of the different orbits; the various changes which the elements of these orbits undergo; the periods and extent of the evagations of the bodies themselves from their mean places; and, lastly, the conservatory principles which ensure the stability of the system, and prevent any unlimited departure from its actual state. It is only with respect to this department of astronomy that the theory can be said to be perfect. The fundamental conditions are simple, and all supplied by observation; and the phenomena are in consequence accurately represented by the analytical equations. But at the surfaces of the planets the law of attraction is modified by various causes, of which the effect cannot be exactly appreciated; and hence the phenomena are less accessible to analysis. Their figures, for example, depend on their initial state, and the law of their density; with respect to which, we can only make arbitrary assumptions—and the motions of their axes of rotation are modified by their figures. For these reasons, the determination of the figures and rotation of the celestial bodies is attended with great and particular difficulties, to the solution of which the most illustrious analysts of the present age have devoted their efforts; and this branch of the theory of gravitation has in consequence received vast improvements since the publication of the *Mécanique Céleste*.

In the development of the planetary theory, Mrs. Somerville has derived great assistance from the *Théorie Analytique du Système du Monde* of Pontécoulant, a recent work of very considerable merit. Though grounded entirely on the *Mécanique Céleste*, the demonstrations in this treatise are occasionally original; while, by a better arrangement, and the adoption of a more uniform method of investigation, they are in numerous cases greatly simplified, without being rendered less general. The mathematical sciences must have undergone some considerable revolution before the theories of physical astronomy can be exhibited in a form much superior to that in which they appear in the work of Pontécoulant.

In a 'Preliminary Dissertation,' extending to seventy pages, Mrs. Somerville has collected and detailed, in a very interesting manner, most of the striking facts which theory and observation have made known respecting the constitution of the universe. This discourse is not indeed strictly confined to the subjects which are discussed in the subsequent part of the work; yet it is not too excursive, if designed as an introduction to the study of the *Mécanique Céleste*. It is calculated to give us a very high opinion of the industry and scientific attainments of its author; as it displays a correct and intimate acquaintance, not only with theoretical astronomy, but with the whole range of physical science, and the best and most recent works which treat of it. The diction, though occasionally deficient in accuracy and precision, is easy, flowing, and

perspicuous; and the topics selected are among the most interesting that science offers to contemplation. The whole is eminently calculated to inspire a taste for the pleasures and pursuits of science; and to promote a desire to penetrate the recesses of that sublime geometry which presides over the motions, and determines the forms and distances, of the planetary bodies. We will quote a few sentences to give a specimen of the style, and the author's opinions on a subject of some moment—the degree of mathematical acquirement required to enter with advantage on the study of the analytical theory of the world. She will be admitted to be no incompetent judge.¹⁶

'The heavens afford the most sublime subject of study which can be derived from science: the magnitude and splendour of the objects, the inconceivable rapidity with which they move, and the enormous distances between them, impress the mind with some notion of the energy that maintains them in their motions with a durability to which we can see no limits. Equally conspicuous is the goodness of the great First Cause in having endowed man with faculties by which he can not only appreciate the magnificence of his works, but trace, with precision, the operation of his laws, use the globe he inhabits as a base wherewith to measure the magnitude and distance of the sun and planets, and make the diameter of the earth's orbit the first step of a scale by which he may ascend to the starry firmament. Such pursuits, while they ennoble the mind, at the same time inculcate humility, by showing that there is a barrier, which no energy, mental or physical, can ever enable us to pass: that however profoundly we may penetrate the depths of space, there still remain innumerable systems, compared with which those which seem so mighty to us must dwindle into insignificance, or even become invisible; and that not only man, but the globe he inhabits, nay the whole system of which it forms so small a part, might be annihilated, and its extinction be unperceived in the immensity of creation.'

'A complete acquaintance with Physical Astronomy can only be attained by those who are well versed in the higher branches of mathematical and mechanical science: such alone can appreciate the extreme beauty of the results, and of the means by which these results are obtained. Nevertheless a sufficient skill in analysis to follow the general outline, to see the mutual dependence of the different parts of the system, and to comprehend by what means some of the most extraordinary conclusions have been arrived at, is within the reach of many who shrink from the task, appalled by difficulties, which perhaps are not more formidable than those incident to the study of the elements of every branch of knowledge, and possibly overrating them by not making a sufficient distinction between the degree of mathematical acquirement necessary for making discoveries, and that which is requisite for understanding what others have done. That the study of mathematics and their application to astronomy are full of interest will be allowed by all who have devoted their time and attention to these pursuits, and they only can estimate the delight of arriving at truth, whether it be in the discovery of a world, or of a new property of numbers.'

The more obvious consequences of the general laws of the universe have been so frequently noticed and illustrated, that it is often extremely difficult to discover by whom they were first remarked. Delambre,¹⁷ in the preface to his *Abrégé d'Astronomie*, takes credit to himself for having always, in speaking of an instrument, a solution, or a formulae, endeavoured to name the author. It is a practice, he observes, too much neglected by the writers of elementary

¹⁶ See *Preliminary Dissertation*.

¹⁷ See note 54, *Preliminary Dissertation*.

works; and the consequence is, that the reader attributes to the author all that he finds in his book, in the same manner as we are led to ascribe to Euclid the theorems he has only transmitted to us. Though it may not be possible, and it is perhaps not necessary, unless where some general principle is involved, to adhere strictly to this practice, yet it is of great importance that the student receive no wrong impressions respecting the history of science; and therefore we cannot help regarding the following as a singularly unfortunate mode of introducing the name of an eminent individual.

‘A fluid, as Mr. Babbage observes, in falling from a higher to a lower level, carries with it the velocity due to its revolution with the earth at a greater distance from its centre. It will therefore accelerate, although to an almost infinitesimal extent, the earth’s daily rotation.’

As well might Mr. Babbage¹⁸ have been quoted as remarking that the tides are caused by the attraction of the moon. The consequence here mentioned is one of those very obvious results of theory, that could not escape the slightest attention to the various circumstances that effect the rotation of the earth. It was stated with great clearness and detail, and without the slightest pretension to originality, by Professor Playfair,¹⁹ in one of the notes to his *Illustrations of the Huttonian Theory*. (Works, vol. i. p. 419.)

Mrs. Somerville’s work contains four books, of which the first, like the corresponding one of the *Mécanique Céleste*, forms a comprehensive and general treatise of Dynamics. On a subject which has been so often discussed by the most eminent mathematicians, we can expect to meet with little novelty or originality; and the principle merit of a new work must consist in the judicious selection and perspicuous arrangement of the materials. The necessary definitions and axioms are given very briefly and clearly in the first chapter; and among the deviations of the methods of Laplace, we cannot forbear noticing the demonstration of the formulæ for the composition and resolution of forces; which Laplace, in order to avoid the assumption of force being proportional to velocity—a thing which cannot be known *a priori*—had deduced immediately from the theory of functions. This demonstration is remarkable; but forms perhaps too great a difficulty at the very commencement of the work. For this reason Mrs. Somerville has rejected it, and returned to the usual demonstration, which depends on the composition and resolution of motion. In this however, she has the countenance of the high authority of Lagrange, who admits that, in separating the principle of the composition of forces from that of the composition of motion, we deprive it of its principle advantages—evidence and simplicity—and reduce it to depend on a mere result of geometrical constructions, or of combinations of algebraical symbols. After the definitions comes the subject of the variable motion of a particle under different circumstances; then the equilibrium and motion of a system of bodies. In these preliminary chapters, the subjects of discussion are the same as those that occur in the *Mécanique Céleste*; and the changes that have been made are chiefly confined to the mode of illustration. The problem of the rotation of a solid body, which occupies the fifth chapter, is of great importance in astronomy, in consequence of its connexion with the theory of the nutation of the earth’s axis, the precession of the equinoxes, and the libration of the moon. The analysis which Mrs. Somerville has given, is the same as that of Pontécoulant, and is sufficiently compact and symmetrical; but the subject is of so difficult a nature, that the general theory cannot be well understood without some special application. The same remarks apply to

¹⁸ See note 14, *Foreword to the Second Edition*.

¹⁹ See note 17, *Preliminary Dissertation*.

the two following chapters, which treat of the equilibrium and motion of fluids. As the theories of the rotation of the earth and of the tides are not comprehended in Mrs. Somerville's work, its unity would, perhaps, have been more perfect, if these last three chapters, which have no subsequent application, had been altogether omitted.

The second book, by far the longest of the four, is devoted to the development of the effects of universal gravitation on the motions and orbits of the primary planets. After a short account of the progress of Physical Astronomy, from Kepler to Laplace, Mrs. Somerville proceeds, in the second chapter, to deduce the Newtonian law of gravity from the three general laws of Kepler.²⁰ These laws form the very basis of the science; and when the differential equations of motion are formed so as to satisfy them, it is an easy consequence that the force which retains the planets in their orbits is directed to the centre of the sun, and varies in the inverse proportion of the distance of the attracted body from that centre. The most obvious verification of this important result is afforded by the motions of the moon; for the action of terrestrial gravity, which at the surface of the earth causes a body to fall through $16\frac{1}{11}$ feet in the first second of time, being assumed to diminish according to the above law, would cause a body at the distance of the moon to fall through a space which is exactly equal to the moon's deflection from the tangent to her orbit in the same time. All this is explained exactly in the same manner as in the *Mécanique Céleste*.

Having deduced from data furnished by observation the law of the force which regulates the motions of the celestial bodies, it becomes necessary to invert the process, and to form the differential equations of motion on the hypothesis, that all bodies of the solar system attract one another with forces varying directly as their masses, and inversely as the squares of their mutual distances. The equations given in the *Mécanique Céleste* are of the utmost generality; being applicable not only to the law of force which prevails in the solar system, but to any law of attraction which is capable of being expressed in a function of the distance. But it is in the integration of these equations that the real difficulty of Physical Astronomy consists; and this difficulty all the ingenuity of the greatest analysts, and all the resources of the most refined science, have hitherto been unable to overcome. It is only by restricting the hypothesis to particular cases that we can obtain even approximate solutions. The particular constitution of the planetary system fortunately affords considerable facilities in this respect; and by permitting us to decompose it into partial systems, and to estimate successively the influence of the different bodies, enables us to obtain results which it would be impossible to arrive at if it were necessary to compass the general problem, and to consider simultaneously all the causes of perturbation. In the first place, though each planet sustains the action of a multitude of forces, yet its motion is chiefly regulated by the predominant influence of the sun, in comparison of which the attraction of any other body in the system, or even the united force of all of them, is extremely small. In the next place, the planetary orbits differ very little from circles, and are inclined at very small angles to the plane of the ecliptic; and on these accounts the series which express the perturbations converge much more rapidly than would be the case if the orbits were more eccentric, and the inclinations considerable. Lastly, the figures of the planets differ so little from spheres, that at their distances the influence of the figure of the disturbing body entirely disappears, and they attract one another as if their whole mass were united in a point at the centre of gravity. These considerations essentially contribute to diminish the difficulties of the calculus.

²⁰ See note 3, *Preliminary Dissertation*.

The simplest hypothetical case to which the equations of motion can be applied, is that of a planet obeying the sun's force, and undisturbed by the action of any other body. to this case Mrs. Somerville proceeds in the fourth chapter. The integration, as is well known, gives a line of the second order; the elements of the curve being represented by the arbitrary constant quantities introduced in the double integration. The development of the expressions thus obtained, gives the whole theory of the elliptic motion. In these elementary discussions, the very brief indications of Laplace have been considerably expanded. The subject admitted of no novelty; but the different formulae for finding the radius vector, the eccentric and true anomalies in terms of the mean anomaly, the true and projected longitudes in terms of the mean longitude, the position of the orbit, &c., are demonstrated with much perspicuity and elegance.

After a first approximation to the true path of a planet has been obtained, on the supposition that it obeys the sun's force alone, it is necessary to pass to the infinitely more difficult problem of the perturbations; and to determine how far the previous results are modified by the attraction of the other bodies belonging to the system, This is the famous problem which was begun and prosecuted with so much vigour by the emulous rivalry of the greatest mathematicians of the last century, — Clairaut,²¹ d'Alembert,²² and Euler²³; and of which the more complete solution has conferred unfading glory on the names of Lagrange²⁴ and Laplace.²⁵ A solution in finite terms is indeed impossible; but the approximations have now been carried so far, that the tables computed from theory, give the places of the planets with a precision that rivals observation.

With a view to facilitate the investigation of this intricate subject, geometers have classed the perturbations under two distinct heads; and the distinction does not depend merely on a difference in the form of the analytical expressions, but on certain physical considerations, which may be easily explained. Let us suppose for an instant the planetary orbits to be invariable in form and position. It is evident that the effect produced by the action of one planet on another, must depend on their relative positions in respect of the sun; for the action of the first planet on the second, may either conspire with the sun's attraction, or oppose it; and it can only cause a variation in the longitude, latitude, or distance of the disturbed planet. Now, this disturbing action will always produce the same effects when the two planets occupy the same positions in respect of the sun, which happens after a certain determinate period of time depending on their relative motions. The relative positions of the planets are technically called their configurations, and are consequently periodic; because after a certain determinate time, the same configurations are again restored. Jupiter, for example, performs his sidereal revolution in about twelve years, and Saturn in nearly twenty years; and at the end of sixty years, therefore, these two planets will be again found nearly at the same points of their orbits, and have the same situation relatively to the sun. If they occupied the same positions exactly, and were disturbed by the influence of no other body, the circle of changes depending on their configuration would then be complete. But the orbits themselves are not fixed; on the contrary, they undergo a continual variation, both in respect of form and position. The transverse axes are slowly revolving on the planes of the orbits; the eccentricities are gradually changing; so also are the inclinations and the position of the nodes relatively to an immovable plane. Now these variations in the forms and positions of

²¹ See note 14, *Bk. III, Chap. II.*

²² d'Alembert, Jean Le Rond, (1717-1783), a pioneer in the study of partial differential equations.

²³ See note 6, *Bk. I, Chap. II.*

²⁴ See note 16, *Preliminary Dissertation.*

²⁵ See note 4, *Foreword to the Second Edition.*

the orbits, give rise to a second class of inequalities, depending not on the configuration of the planets, but on the relative positions of the major axes, or the configurations of the orbits. Like those of the former class, they are periodic; but their periods are vastly longer, as the revolutions on which they depend are incomparably longer than the revolutions of the planet. The axis major of the earth's orbit accomplishes a revolution in 109,770 years, and that of Jupiter in 197,561 years; hence an idea may be formed of the time required to complete the cycle of inequalities depending on such slow motions. On this account the inequalities depending on the positions of the orbits are called *secular*. Some of them were detected by comparing observations made at distant epochs; but in general they escape observation, by the slowness of their evolutions.

The particular analytical procedure by which the computation of the various inequalities is brought within the power of the calculus, is particularly deserving of attention. It is founded on the supposition that the elements of a planet's orbit are constantly varying; or that the planet only continues to describe the same ellipse during an infinitely small portion of time. The arbitrary quantities, therefore, which enter into the integrated equations of motion, and represent the elements of the elliptic orbits, are considered as variable; their variations being expressed in terms of the partial differentials of the perturbing force. The germ of this method, as of many others of the first importance in analysis, is due to Euler; but the complete theory properly belongs to Lagrange, by whom, after many successive modifications and improvements, it was reduced to its last degree of elegance and generality. As it now stands, the theory of the planetary perturbations is reduced to the integration of a system of linear equations, in which the differential of each elliptic element is expressed by the partial differentials of the perturbing force, multiplied by the element of the time. The great advantage of the method consists in its affording the means of exhibiting, under a single point of view, all the effects arising from the reciprocal actions of the planets, whether secular or periodic, either in their motions of translation or rotation; as well as the derangements that would be produced by a resisting medium, or any other disturbing cause whatever. When the combined action of a great number of forces is to be calculated, there is no more efficient method than this in the whole range of analysis.

At the time the first two volumes of the *Mécanique Céleste* made their appearance, the theory of the variation of arbitrary constants had not reached the degree of perfection it has since attained. In the second volume, Laplace gave expressions for the variations of the eccentricity, the inclination of the orbit, and the longitude of the nodes only; the expressions for the variations of the of the remaining two elements—namely, the longitudes of the perihelion and epoch—are given in the supplement to the third volume. By partially adopting the method of Lagrange, and taking advantage of the more recent discoveries of Poisson,²⁶ who also has essentially contributed to the perfection of this theory, Pontécoulant has succeeded in rendering the subject greatly more perspicuous; and Mrs. Somerville has judiciously availed herself of the labours of Pontécoulant. The investigation commences with the demonstration of a formula due to Lagrange, for expressing the partial differential of one of the elliptic elements of an orbit, in a linear function of the infinitely small variations of that element, multiplied by certain combinations of the partial differential of the perturbing force, taken with respect to the rectangular co-ordinates of the troubled body. The formula is next applied to the variation of the differential elements in succession, without laying down any restricted hypothesis as to the magnitude of the eccentricities and inclinations; after which, the modifications are pointed out which the expressions receive in consequence of the smallness of the eccentricities and

²⁶ See note 1, *Bk. I, Chap. VI.*

inclinations of the actual orbits of the planets. All these expressions involve the differential of the function which expresses the perturbing force; the expansion of which into a series, and the determination of the coefficients of its several terms, occupy the remainder of the fifth chapter. This development depends ultimately on that of the irrational factor $(a^2 + 2ab\cos A + b^2)$ —into a series of cosines of the multiples of the angle A ;—a subject which seems first to have engaged the attention of Euler in his *Memoir on the Inequalities of Jupiter and Saturn*, and which, on account of its great importance in the theory of the planetary perturbations, has been frequently treated by mathematicians. In certain cases—that is to say, when the ratio of the distances of the disturbed and disturbing planet is very small—the expanded series converges with sufficient rapidity; but when that ratio approached nearer to unity, as happens in the case of Venus and the Earth, or Jupiter and Saturn, the series converges slowly, and it becomes necessary to have recourse to particular artifices in order to obtain the values of its different terms. The labour of computation is, however, greatly facilitated in consequence of a curious relation discovered by Euler to subsist among the terms; which is such, that when any two of them are found, all the others can be determined in a function of these two; hence the difficulty is confined to the determination of the first two terms, and this has been effected in a great many different ways. Mrs. Somerville has taken the development exactly as it is given by Pontécoulant; and though in principle the same, it has the merit of being considerably simpler than that of Laplace.

It is in the development of the function which expressed the perturbing force, that the two distinct sets of terms arise which respectively represent the periodic and secular inequalities. One part of the expanded function consists of terms having for their argument the sines or cosines of the mean motion and its multiples; while the other terms are entirely independent of the mean motion, being merely functions of the elements of the orbits, and their combinations. The determination of this last set of terms is of the utmost consequence in theoretical astronomy; for if they were susceptible of indefinite increase with the time, the forms of the orbits and the periods of revolution would, in the course of ages, be entirely altered, and the stability of the planetary system destroyed. To this subject Mrs. Somerville addresses herself in the sixth chapter, and examines in detail the terms, independent of the time, which are contained in the variations of each of the elliptic elements.

Of all the elements of a planet's orbit, the axis major is that of which the variations are the most important, on account of the relation subsisting between the mean distance and the mean motion. Accordingly, the efforts of geometers have been particularly directed to this subject, and their successive discoveries distinctly mark the progress of analysis. The first Memoir which Laplace presented to the Academy of Sciences, in 1773, contained the very important discovery that the mean distances and mean motions include no secular inequality, or term increasing with the time, when the approximations are carried to the third powers of the eccentricities and inclinations, and regard is had only to the simple powers of the disturbing force. Stimulated by this remarkable result, Lagrange undertook the investigation of the same subject, and demonstrated, in the Berlin Memoirs of 1776, that on having regard only to the first power of the disturbing force, the differential expression of the major axis can include no term increasing indefinitely with the time, to whatever order of terms the approximations may be carried with regard to the eccentricities and inclination; unless indeed there should exist a commensurable ratio between the mean motions of the disturbed and disturbing planet. Such a condition, however, does not exist in the planetary theory; and therefore the greater axes and mean motions are only susceptible of periodic inequalities depending on the configurations of the planets, and of which the limits may be assigned. But although this approximation is

sufficient in regard to the other elements, it is necessary in the case of the major axis to proceed a step farther, and to have regard to the terms depending on the second powers of the disturbing force; because such terms, though multiplied by the squares of the masses, being expressed by second differentials, may acquire in the double integration very small divisors; in consequence of which their values become comparable to those which, in the case of the other elements, depend on the first powers of the masses, and are given by a single integration. Laplace showed, in the sixth book of the *Mécanique Céleste*, that the mean motions of Jupiter and Saturn are not altered by their great inequalities, even when regard is had to the squares of the disturbing forces; but Poisson had the merit of first demonstrating generally, that the terms depending on the squares and products of the perturbing force can introduce no secular inequalities into the expressions of the greater axes or mean motions. This was an important and necessary extension of the great discovery of Lagrange. Mrs. Somerville refers us to a recent paper in the Philosophical Transactions, in which the demonstration of the permanency of the mean motions is said to be carried to *all* the powers of the disturbing masses. This result, if well verified, must be of great interest in regard to analysis, though it is fortunately of no importance to astronomy.

From the consideration of the major axes, Mrs. Somerville passes to that of the other elements of the orbits. In respect of these elements the stability of the system is equally assured as in the case of the mean motions. They are not, indeed, like the mean motions, exempted from the influence of secular perturbations; but their inequalities, though independent of the configurations of the orbits, are nevertheless subject to the law of periodicity, and can never exceed certain small limits. These consequences result from certain relations that subsist among the elements of all the orbits, and limit the increase of their variations. Thus the eccentricities, though subject to slow variations, can never entirely disappear, but must always continue to vibrate about a mean state; subject to the remarkable condition, that 'the sum of the squares of the eccentricities, multiplied by the masses of all the bodies of the system, and by the square roots of the axes of the orbits, remains always the same constant magnitude.' The same condition must be fulfilled with respect to the inclinations of the orbits to a fixed plane. The variations of the longitude of the epoch are extremely important on account of their influence on the mean longitudes of the planets. Theory show that they exist; but they are altogether insensible to observation in the case of the planets. Even in the case of Jupiter and Saturn, the two planets whose mutual perturbations are the most remarkable, the variation of this element amounts to less than the 60th part of a second in a century, and requires no less than 70,414 years to complete its period. The motions of the perihelia are the only elements to the variations of which no limit has yet been assigned; but it is certain that they must always continue to vary with extreme slowness, as they do at the present time.

After having discussed those terms in the variations of the different elements which are independent of the mean motions, and give the secular inequalities, the next step is to return to those which depend on the sines and cosines of the mean motion, and give the periodic inequalities. These being of a simpler kind, had been for the greater part determined by peculiar considerations, before the general method of deducing the inequalities of both kinds from the variations of the elliptic elements had been discovered by Lagrange; but it is of great importance to the progress of the science, that, as all the inequalities are occasioned by the same physical cause, they should also be all comprehended in the same general analysis, and deduced by uniform methods. In reference, however, to the ultimate object of astronomy, that of determining the positions of the planets in space, it is not material to know particularly the alterations which each of the elements of an orbit undergoes; for the periodic variations always remain very small,

and have only a transient effect on the orbits. It is sufficient to know the amount of their combined influence on the places of the planets, or the three polar co-ordinates by means of which their positions are fixed, viz. the distances, longitudes and latitudes. Lagrange's method of obtaining these elements in the disturbed orbit, is at once simple and elegant. In the case of elliptic orbits, the radius vector, the longitude and latitude are expressed by series which proceed according to the ascending powers of the eccentricities and inclinations; in these series, therefore, he substitutes for the elliptic elements the same elements corrected for the periodic and secular variations found from the general formulæ; and thus obtains correct expressions for the radius vector, the longitude and latitude of the troubled orbit. In this manner the position of the planets, at every instant, may be computed by known rules. Mrs. Somerville has given the development of this method, in the seventh and eighth chapters, from Pontécoulant. The original may be found in Lagrange's Memoirs on the periodic variations of the *Motions of the Planets*, published in the *Berlin Memoir* for 1783.

The method of Lagrange here referred to, though extremely ingenious and important in respect of analysis, is not that which leads most directly to the determination of the periodic variations. When the secular inequalities are left out of view, and particularly when it is not required to extend the approximations beyond the simple powers of the eccentricities and inclinations, the easiest method is to deduce the periodic inequalities directly from the differential equations of the orbit; for in this way we arrive at once at the variations of the longitude, latitude, and radius vector. This method is given in the ninth chapter, and the approximations are carried to the third powers of the eccentricities and inclinations.

Before proceeding farther in this analysis, we cannot avoid expressing our regret that Mrs. Somerville has not given any preliminary explanation of the peculiarities of the analytical methods she exposes, or the principles on which they are founded. In the eighth chapter, the variations of the polar co-ordinates of a planet are given according to Lagrange's method. In the following chapter, a 'second method of finding the perturbations of a planet in longitude, latitude, and distance,' is announced; and the reader, without being informed in what respect the first method is insufficient, or how the second differs from the first, or of any circumstances that can render a second method necessary, is hurried into the midst of an intricate investigation, the uses and object of which he is left to infer, as well as he can, when he arrives at the end of the calculus. This deficiency, or rather entire absence of all explanations or discussion of the peculiarities of the different methods and analytical processes made use of, is the greatest defect of the work, and cannot fail to render its perusal more discouraging and far less instructive than it ought to be, considering the perspicuous arrangement of the subjects. Half the difficulty of a geometrical investigation may be said to be overcome when a distinct perception has been acquired of the object to be attained, and the route to be followed.

Among the terms of the series which expresses the mutual perturbations of two planets, there are some into which the difference between certain multiples of the mean motions enters by integration as a divisor; and if it happens that this difference is very small, or that the mean motions of the two planets are nearly commensurable, such terms, though minute in themselves, may acquire, in consequence of the smallness of their divisors, very considerable values. The mean motions of no two planets in the solar system are exactly commensurable; but those of Jupiter and Saturn approach so nearly to commensurability, that part of the terms belonging to the third and fourth powers of the eccentricities and inclinations, have, in consequence, appreciable values. In the computation of the inequalities of these two planets, therefore, it becomes necessary to push the approximations so far as to include the terms of the fourth order

in respect of the eccentricities and inclinations; and likewise to retain those that depend on the square of the perturbing force. On this account the theory of Jupiter and Saturn forms a peculiar, and as it were, a supplementary case of the problem of the planetary perturbations, the solution of which long baffled the efforts of the first mathematicians. The inequalities of their mean motions are so considerable that they had been discovered by Halley²⁷ from a comparison of observations. Euler had failed in the attempt to connect them with theory; Lagrange only proved that they did not belong to the class of secular inequalities; it was, therefore, for some time supposed that Jupiter and Saturn form an exception to the general principle of the invariability of the mean motions. At length Laplace, with that characteristic sagacity which enabled him on so many other occasions to detect the expression of a physical fact among the mazes of an intricate calculus, discovered the cause of the anomaly, in the near commensurability of the mean motions. The long period of the inequalities in question, namely, 929 years, might easily cause them to be mistaken for secular inequalities. The discovery of their true source and amount, which was necessary to the perfection of theory, has had an important influence on the astronomical tables; the errors of which, in respect of Jupiter and Saturn, hardly now exceed $1\frac{3}{4}$, whereas, not more than twenty years ago, they amounted to a hundred times that quantity.

The theory of Jupiter and Saturn is given in the tenth chapter. We may remark that the computation of the terms depending on the square of the perturbing force is extremely laborious, and that the greatest mathematicians of the present day are not agreed with respect to their exact numerical values.

In the three following chapters Mrs. Somerville discusses the inequalities depending on the ellipticity of the sun, and the action of the satellites, and the data requisite for computing the motions of the planets. The fourteenth chapter is of a very miscellaneous nature,—including the numerical values of the perturbations of Jupiter; remarks on the transits of Venus and Mercury; the perturbations of the Earth, Mars, and the other planets; remarks on the atmospheres of the planets; on the spots and motion of the sun; on the zodiacal light; the influence of the fixed stars in disturbing the system; and the construction and correction of the astronomical tables. This concludes the planetary theory.

The third book belongs to the lunar theory. The problem of finding the lunar perturbations is essentially a problem of the same nature with that of finding the perturbation of a planet; but on account of the great eccentricity of the lunar orbit, and the powerful attraction of the sun, which is in this case the disturbing body, it is necessary to carry the approximations farther than is generally required in the planetary theory. The terms depending on the square of the perturbing force, are not only sensible, but they even double the motion of the lunar perigee; and in computing several of the inequalities, it is necessary to include the fourth, and even the fifth powers of the eccentricity and inclination. It would be extremely difficult to convey any idea of the method employed by Laplace to determine the numerous and complicated inequalities of the moon, without entering into the details of analysis. The lunar theory is certainly the most remarkable portion of the *Mécanique Céleste*, whether we regard it as a mere problem of analysis, or in reference to its important applications in practical astronomy. It unites in itself, says Laplace, all that can give value to discovery—grandeur and utility in the object, fecundity of results, and the merit that attaches to the conquest of great difficulties.

The most remarkable of the lunar inequalities are periodic, and occasioned by the action of the sun; and the difficulty of determining them is chiefly owing to the slow convergence of the

²⁷ See note 55, *Preliminary Dissertation*.

series. But besides the perturbations which the moon sustains directly from the sun and the planets, her motions are greatly complicated, from the circumstances of her not moving round a fixed centre like the planets, but round a body which is itself in motion, and the elements of whose orbit partake of the general disturbance. All the inequalities that effect the motion of the earth are attended with corresponding effects on the motion of the moon, and are even more sensible in proportion as the moon is further from the common centre of gravity. The variation of the eccentricity of the earth's orbit, for example, introduces secular inequalities into each of the three lunar co-ordinates, namely, the parallax, latitude, and longitude. On the parallax, however, its influence is so small as to be insensible to observation. On the longitude its effects are perceptible, as it occasions that acceleration of the moon's mean motion which had been detected by a comparison of ancient with modern eclipses, and of which the physical cause was only discovered by the powerful analysis of Laplace. On the latitude its effects are manifested in a retrograde motion of the nodes. It effects in a still more sensible degree the motion of the perigee, which becomes slower from century to century. These three inequalities are related to one another in such a manner, that if the variation of the mean motion be called 1, the variation of the nodes is .734, and of the perigee 3, very nearly. The acceleration of the mean motion amounts to $10''.2$ in a century: and it is remarkable, that while the mean motion continues to be accelerated, the motion of the perigee and nodes is retarded.

When these three inequalities shall have developed in the course of ages, and their values determined by a long series of observations, they will lead to a more accurate knowledge than we yet possess of the extent and period of variation of the eccentricity of the terrestrial orbit. This is occasioned principally by the disturbing influence of Mars and Venus; hence if the variations of the earth's eccentricity were correctly known, we should be able to assign an accurate value of the masses of these two planets. It is a striking instance of the intimate dependence of all the phenomena of the planetary system on one another, that by merely observing the moon the astronomer is enabled to determine the quantity of matter in Mars and Venus; and yet science reveals many more wonderful secrets.

Another source of inequality peculiar to the lunar motions, is the non-sphericity of the earth. On account of the moon's proximity, the compression of the earth has a sensible influence on her motions, and occasions two inequalities, to compute which it is necessary to have recourse to the theory of the attraction of spheroids. One of them has for its argument the longitude of the moon's node; the other is an inequality of the motion in latitude, depending on the moon's mean longitude. These two inequalities, determined from a great number of observations, concur in giving an ellipticity of $\frac{1}{305}$ nearly, agreeing in a surprising manner with the results obtained from the measurement of terrestrial degrees, and observations of the pendulum. In all probability they give the most accurate determination of the figure of the earth, being independent of local disturbance. From this result we are enabled to deduce some inferences respecting the interior constitution of the earth. It was demonstrated by Newton that a fluid mass of homogeneous matter, revolving with the same velocity as the earth, would acquire a compression of $\frac{1}{230}$; hence the earth is not homogeneous, but increases in density from the surface towards the centre. Again, if any difference exists in the form or constitution of the two terrestrial hemispheres, it would give rise to a lunar inequality proportional to the cosine of the longitude of the perigee, augmented by twice the longitude of the moon's orbit. Observation has failed in detecting any inequality of this sort; there is consequently no sensible difference of form or constitution in the two hemispheres. It is also to the attraction of the earth that we must refer

the rigorous equality that subsists between the mean motions of rotation and revolution of the moon, in virtue of which the same hemisphere is constantly turned towards the earth.

Laplace has likewise investigated the effect that would be produced on the lunar motions by the resistance of a gaseous medium of great rarity occupying the planetary spaces; the existence of which many phenomena, particularly the propagation of light, render extremely probable. The immediate effect of the resistance of a medium on a planet would be to diminish the tangential velocity, and consequently the centrifugal force. This would allow the action of gravity to draw the moon nearer the earth, and cause an acceleration of her angular velocity or mean motion. A similar effect would be produced on the earth, and the other planets; but the effect on the moon would be a hundred times more sensible than on the earth. But the observed acceleration of the moon is perfectly explained from the theory of attraction; and, therefore, if the regions of space are filled with an elastic medium, it must be so rare as to offer no resistance to the planets or satellites. Additional interest has lately been given to this question, from the circumstance that Encke's comet²⁸ seems to have an accelerated motion, which it is difficult to explain on any other hypothesis; but this body must be observed in many of its future revolutions, before a conclusion of so much importance can be considered as well established. In the mean time, it has been computed by Mazotti, that, if the phenomenon in question is caused by the resistance of an ethereal medium, its rarity must be 360,000 millions of times greater than that of atmospheric air.

Before the true cause of the moon's acceleration was discovered, it had been suggested that the phenomenon might be occasioned by the successive transmission of gravity from the earth to the moon. Laplace also investigated the consequences of this hypothesis, and found that, in order to produce the observed acceleration, the velocity with which gravity is transmitted must be 42 millions of times greater than that of light. But neither the resistance of an ether, nor the successive transmission of gravity, can produce the secular variation of the lunar nodes and perigee; these two inequalities consequently afford of themselves the most convincing proof, that all the celestial motions are performed in obedience solely to the Newtonian law of gravity.

'It is evident,' says Mrs. Somerville, 'that the lunar motions can be attributed to no other cause than the gravitation of matter: of which the concurring proofs are the motion of the lunar perigee and nodes; the mass of the moon; the magnitude and compression of the earth; the parallax of the sun and moon, and consequently the magnitude of the system; the ratio of the sun's action to that of the moon, and the various secular and periodic inequalities in the moon's motions, every one of which is determined by analysis on the hypothesis of matter attracting inversely as the square of the distance; and the results thus obtained, corroborated by observation, leave not a doubt that the whole obey the law of gravitation. Thus the moon is, of all the heavenly bodies, the best adapted to establish the universal influence of this law of nature; and from the intricacy of her motions, we may form some idea of the powers of analysis, - that marvelous instrument, by the aid of which so complicated a theory has been unraveled' -

The satellites of Jupiter furnish another case of the problem of attraction, having also its peculiar difficulties. Ever since the discovery of these bodies by Galileo,²⁹ they have been objects of great interest both to the practical astronomer and to the geometer; to the former, on account of their connexion with the problem of the longitude; and to the latter, on account of the

²⁸ See Art. 618, Bk. II.

²⁹ See note 1, Introduction.

difficulty of submitting their intricate motions to analysis. They exhibit, as it were, a miniature representation of the solar system, in which, by reason of the promptitude of their revolution, all the inequalities arising from their reciprocal action pass through their cycle of changes in comparatively short periods of time. If the figure of the primary planet could be neglected, the problem would be one of five bodies; but the ellipticity of Jupiter has a very powerful influence on the motions of the satellites; and for this reason it becomes necessary to have regard not only to his figure, but also to the inclination of his equator and ecliptic, and the position of his nodes. hence the problem becomes extremely complicated, and is embarrassed with the details of numerous computations. Lagrange was the first who ventured to grapple with it in all its difficulty; but his solution, as is remarked by Delambre,³⁰ though a wonderful display of his power of analysis, contributed little or nothing towards the amelioration of the tables of the satellites; and it is remained for Laplace to complete the theory, and to substitute formulae rigorously deduced from the differential equations of motion for the empirical equations from which the eclipses has been computed. This theory is contained in the eighth book of the *Mécanique Céleste*, and may be regarded as nearly perfect. All the inequalities have corresponding expressions in the theories of the moon and planets. the mean motion of the first added to twice the mean motion of the third, is rigorously equal to three times the mean motion of the second; and it is extremely remarkable that the secular inequalities of their mean motions, and their motions of rotation, are also subject to the same law.

Before the solution of the problem of the satellites can be rendered of any avail to astronomy, it is necessary to assign values from observation to the quantities which analysis leaves indeterminate. These are, the six elliptic elements of each orbit, the mass of each of the satellites, the ellipticity of Jupiter, the inclination of his equator to his ecliptic, and the position of his nodes—in all thirty-one. This most laborious task was undertaken by Delambre, who computed all the recorded eclipses, amounting to six thousand; and the tables, subsequently improved by Bouvard, now give the positions of the satellites, and the time of the eclipses, more accurately perhaps than direct observation. Great importance was formerly attached to the theory of the satellites, on account of the easy means their eclipses furnish of determining the difference of terrestrial meridians; but since the lunar tables have been brought in a manner to perfection, it has lost much of its interest in practical astronomy. The difficulty of determining the exact instant at which a satellite enters the shadow of Jupiter is such, that it is not rare to find two observations of the same eclipse differing by thirty seconds of time, even in the case of the first satellite, whose motions are by far the most rapid. Various causes concur to produce this uncertainty; among which may be mentioned the ill-defined contour of the shadow of the planet, which renders a satellite longer visible to a good eye, or in a good telescope. Much depends also on the position of Jupiter with respect to the sun, or of the satellite in respect of Jupiter; and perhaps something also to a difference in the physical state of the surface of the satellite, which may render one side of it better fitted to reflect the sun's light than the other. These uncertainties disappear when a great number of observations can be combined; but when one or two observations only can be procured, the certainty of the result is not to be put in comparison with that of one furnished by the lunar occultation of a fixed star.

Mrs. Somerville has treated the subject of the satellites, nearly in the same manner as the lunar theory. The method of Laplace is closely followed, and the equations for the greater part transcribed without alteration. The arrangement is in some respects different; but this is a matter of inferior moment; and as the explanations of the analytical operations are somewhat

³⁰ See note 54, *Preliminary Dissertation*.

compressed, we apprehend a student would have still more difficulty in mastering this intricate theory from Mrs. Somerville's exposition, from that of the *Mécanique Céleste* itself.

With regard to the satellites of Saturn and Uranus, the difficulty of observing them is so great, that it is only in some instances that their mean distances and periodic times have been ascertained. Their theory can never be of any practical use; but it is interesting to trace the effects of gravitation among those remote and minute bodies, whose existence is but recently known. and which are only discernible in the most powerful telescopes.

The subjects which occupy the remainder of the *Mécanique Céleste* are of great extent and importance; but, in the progress of analysis, other methods have been discovered, by which they may be treated with greater advantage. For example, the methods which Laplace has given for determining the orbits of comets, though stamped with all the characteristics of his powerful mind, is not the most convenient in practice; because it assumes the numerical values of the first and second differential coefficients of the longitude and latitude as functions of the time to be exactly known from observation. The determination of these quantities, however, is often a matter of much difficulty; and when only a few observations can be obtained, as happens in the greater number of cases, cannot be relied on. In other respects, the theory of comets is imperfect; for mathematicians have not been able to represent the perturbations these bodies sustain from the planets by formulae which embrace an indefinite number of revolutions. This may still be considered a *desideratum* in physical astronomy,—the more to be regretted, as the interest attaching to the cometary theory has been vastly increased by recent discoveries. The mathematical theory of the figure of the earth has taken an almost entirely new form in the hands of Ivory and Poisson; so that this portion of the *Mécanique Céleste* is now interesting only as an exercise of analysis. The theory of the tides requires to be entirely recast. Mrs. Somerville has, therefore, wisely selected that department of Physical Astronomy which, in consequence of the degree of perfection it has attained, is most likely to retain its present form. We take leave of her work with the renewed expression of the admiration we have experienced in perusing the proofs which it so strongly affords of high and rare attainments; and of gratitude for what she has done with a view to diffuse the knowledge of those sublime truths which mathematical analysis has so largely revealed.

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THE close of the last century witnessed the successful termination of that great work, commenced by Newton, and prosecuted by a long succession of illustrious mathematicians, by which the movements of the planetary system were reduced under the expression of dynamical laws, and their past and future positions with respect to their common centre and to each other, rendered matter of strict calculation. A wonderful result, which will forever form a principal epoch in the history of mankind, was at length arrived at in the announcement of the fact, that a brief and simple sentence, intelligible to a child of ten years of age, accompanied with a few determinate numbers capable of being written down on half a sheet of paper, comprehends within its meaning the history of all the complicated movements of our globe, and the mighty system to which it belongs—the mazy and mystic dance of the planets and their satellites—'cycle on epicycle, orb on orb'—from the earliest ages of which we have any record, nay, beyond all limits of human tradition, even to the remotest period to which speculation can carry us forward into futurity. By the announcement of this law and the establishment of these data, an indefinite succession of events is thus combined into one great fact, and may be considered as a single feature in creation, independent of the lapse of time, and registered only in the unproductive annals of eternity.

In the course of the investigations which have terminated in this result, another fact, of a no less high and general order, has come to light of which Newton could have formed no anticipation, that, namely, of the stability of our system, and the periodic nature and restricted limits of its fluctuations, which preclude the possibility of such deviations from a mean or average state as may lead to the subversion of any essential feature of that happily balanced order which we observe at present to subsist in it. This noble theorem forms a beautiful and animated comment on the cold and abstract announcement of the general law of gravitation. A thousand systems might have been formed of which the motions would, for a time have been regular and orderly enough, but which would either have ended in a collision of parts subversive of the original conditions, or would pass through a succession of phases or states, endless in variety, among which some would be found no less incompatible with life than such collisions themselves—whether from extreme remoteness or proximity of the source of light and heat, or from violent and sudden alternations of its influence—or in which, at all events, that beautiful and regular succession of seasons—that 'grateful vicissitude' we admire and enjoy, and those orderly and established returns of phenomena which afford at once the opportunity and the inducement to trace their laws, would have been wanting; while in their place might have reigned a succession of changes reducible to no apparent rule; variety without progressive improvement; years of unequal length and seasons of capricious temperature; planets and moons of portentous size and aspect, glaring and disappearing at uncertain intervals, and every part of the system wearing the appearance of anarchy, though, in fact, obeying, to the letter, the same general law of gravitation, which must yet have for ever remained unknown to its inhabitants.

³¹ See note 64, *Preliminary Dissertation*.

Among infinite systems equally possible, such, we have no reason to doubt, might exist—but our own is not, nor can it ever, in its own natural progress, pass into such a one. In the choice of its arbitrary constants, (to use the language of geometers,) in the establishment of the relations of magnitude, speed, and distance of its parts, such a case is expressly provided against. In the circulation of its members all in one direction—in the moderate amount of the eccentricities and inclinations of all the planetary orbits, and the extremely small ones of those of its more important bodies, but more especially in the mode in which the general system is broken up into several subordinate ones, and in the individual attachment and allegiance of each member to its immediate superior, we must look to the safeguards of this glorious arrangement.

This last mentioned condition may require some illustration. Had the Earth and Mars, for instance, formed a binary combination separated by an interval no greater than the moon's actual distance from the earth, there is no doubt that such a double planet might have continued to circulate round the sun nearly as the earth and moon do at present. But with such a combination the moon could not have coexisted, without a complete breach of her law of regular periodicity. Its path would alternately by one and the other of its great equipollent centres, whichever, for the moment, occupied the most advantageous position; and should its primitive velocity be so adjusted that it could neither throw itself to a sufficient distance from both to escape from the influential attraction of either, and become a separate planet, nor attach itself so closely to one of them as to be carried about it as a mere appendage, it must continue to wind, for ever, an intricate and sinuous course around and between them, in which occasional collision with one or other would, by no impossible or improbable contingency, afford a tragic epoch in the history of so ill-adjusted a system.

It is, moreover, well worthy of remark, that the mode in which the stability of our system is accomplished is by no nice mathematical adjustment of proportions,—no equilibrated system of counterpoises satisfying an exact equation, and which the slightest deviation in any of the data from its strict geometrical proportion would annul. Such adjustments, it is true, are not incompatible with the law of gravitation, even in a system composed of several bodies. Geometers have demonstrated, for example, that three or even more bodies, exactly adjusted in their weights and distances, and in the velocities and directions in their motions at any one instant, might forever describe conic sections about each other, and about their common centre of gravity. But without supposing any such adjustment of the weights and distances of the members of a system subjected to the law of gravitation, and taking them as they are actually in our own, there is yet another supposition in which the absence of secular perturbation might have been ensured,—that, namely, in which the planetary motions should be performed all in one plane, and all in perfect circles about the sun,—realizing, in fact, the old Aristotelian notion of celestial movements, all which he considered to be of necessity exactly circular. We do not remember to have seen any mention made of the possibility of this case. It follows, however, immediately, from the general proposition demonstrated by Lagrange and Laplace, which establishes an invariable relation among the eccentricities of any number of perturbed orbits; viz. that the sum of the squares of all the eccentricities, each multiplied by an invariable coefficient, is itself invariable, and subject to no change by the mutual action of the parts of the system. For it is evident, that had the orbits been all originally circular, or if at any one instant of time each of the eccentricities were, by some external agency, destroyed, so as to render these orbits at once all circles, after which the system should be abandoned to its own reactions, the sum in question would also vanish at that instant, and therefore at every subsequent instant, which would be

impossible, (since none of the coefficients are negative,) unless each several eccentricity were to remain for ever evanescent *per se*, or each several orbit a perfect circle.

If we depart from the law of gravitation, and inquire whether, under other conceivable laws of central force, a system might not exist essentially and mathematically free from the possibility of perturbation, and in which every movement should be performed in undeviating orbits and unalterable periods, we have not far to search. Newton has himself demonstrated, in his *Principia*, or at least it follows almost immediately from the 89th proposition of his first book and its corollary, that this wonderful property belongs to a law of attractive force in the *direct* proportion of the distance; and, however extravagant such a supposition may appear, if we consent to entertain it as a mere mathematical speculation, it is impossible not to be struck with the simplicity and harmony which would obtain in the motions of a system so constituted. Whatever might be the number, magnitudes, figures, or distances of the bodies composing an universe under the dominion of such a law—in whatever planes they might move, and in whatever directions their motions might be performed—each several body would describe about the common centre of gravity of the whole, a perfect ellipse; and all of them, great and small, near and remote, would execute their revolutions in one common period, so that, at the end of every such period, or *annus magnus*, of the system, all its parts would be exactly re-established in their original positions, whence they would set out afresh, to run the same unvarying round forever.

We may please ourselves with such speculations, and enjoy the beauty and harmony of their results, in the very same spirit with which we rejoice in the contemplation of an elegant geometrical truth, or a property of numbers, without presumptuously encroaching on the province of creative wisdom, which alone can judge of what is really in harmonious relation with its own designs. The stability of our actual system, however, rests on a basis far more refined, and far more curiously elaborate. It depends, as we have before observed, on no nice adjustments of quantity, speed and distance. The masses of the planets, and the constants of their motions, might all be changed from what they are, (within certain limits,) yet the same tendency to self-destruction in the *deviations* of the system from a mean state, would still subsist. The actual form of their orbits are not ellipses, but spirals of excessive intricacy, which never return into themselves; yet this intricacy has its laws, which distinguish it from confusion, and its limits, which preserve it from degenerating into anarchy. It is in the conservation of the principle of order in the midst of perplexity—in this ultimate compensation, brought about by the continued action of causes, which appear at first sight pregnant only with subversion and decay—that we trace the master workman, with whom the darkness is even as the light.

This momentous result has been brought to light slowly, and, as it were, piecemeal. The individual propositions of which it consists have presented themselves singly, and at considerable intervals of time, like the buried relics of some of those gigantic animals which geologists speak of, each, as it emerged, becoming a fresh object of wonder and admiration, proportioned to the labour of its extraction, as well as to its intrinsic importance; and those feelings have at length been carried to their climax by finding the disjointed members fit together, and unite into a regular and compact fabric.

It is our continental neighbours, but more especially to the geometers of France, that we owe the disclosure of this magnificent truth: Britain has taken little share in the inquiry. As if content with the glory of originating it, and dazzled and spellbound by the first great achievement of Newton, his countrymen, with few and small exceptions, have stood aloof from the great work of pursuing into its remote details and general principle established by him. We are far from being disposed to attribute this remarkable supineness to the prevalence of any of

the meaner or more malignant feelings of national pride, prejudice, or jealousy. Some irritation and distaste for the continental improvements might be, and no doubt were, engendered, and, to a certain extent, continued by the controversies which excited so lively a sensation among the contemporaries of Newton; but, on the other hand, it could not have been, at first, reasonably presumed, (what proved afterwards to have been really the case,) that the applicability of Newton's mode of investigation should terminate almost at the very point where he himself desisted from applying it—still less that algebraic processes, which were regarded by him as mere auxiliaries to geometrical construction and demonstration, should be destined to acquire such strength and consistency as to supercede all others, and leave them on record only as scientific curiosities. It is rather to the barrier thrown by our insular situation in the way of frequent personal communication between our mathematicians and those abroad, to the want of a widely diffused knowledge of the continental languages, and to the consequent indifference in the reading part of the public as to the direction which thought was taking, in the loftier regions of its range, in other lands than our own, that we are inclined to refer what cannot but appear an extraordinary defect of sympathy in so exciting a course of discovery. Much, too, must be attributed to that easy complacency with which human nature is too apt to regard progress already made as all that can be made;—which dwells with admiring and grateful satisfaction on achievements performed and laurels won, while it neglects to body forth the possibilities of a yet richer and more glorious future;—suffers a short breathing time to become prolonged into a state of languor and indifference; and consigns to other and fresher aspirants the toil and the reward of penetrating farther into those thorny and entangled thickets of unexplored research which bound our actual horizon, and by the force of habit and repose come at length to hedge in our thoughts and wishes.

Whatever might be the causes however, it will hardly be denied by any one versed in this kind of reading, that the last twenty years of the eighteenth century were not more remarkable for the triumphs of both the pure and applied mathematics abroad, than for their decline, and, indeed, all but total extinction, at home. From the publication of Waring's³² profound but cumbrous *Meditationes Algebraicae*, and Landen's³³ researches on the motions of solids, and his remarkable discovery of the rectification of the hyperbola by two ellipses, we may search our libraries in vain for investigations of the slightest moment in the higher analysis, or, indeed, for any evidence of its abstruser parts being so much as known to our mathematical writers. While the academical collections of Turin, Paris, Berlin, and Petersburg, were teeming with the richest treasures of the analytic art, poured forth with unexampled profusion, our own presented the melancholy contrast of entire silence on all the great questions which were then agitating the mathematical world,—a blank, in short, which the respectable names of Vince and Hellins only served to render more conspicuous.

It was with the commencement of the present century that a sense of our deficiencies, and of the astonishing and disreputable distance to which we had fallen behind the general progress of mathematical knowledge in all its branches, began to make itself felt; but to remedy the evil was more difficult than to discover its existence. Great bodies move slowly. It requires time, where national tastes and habits are concerned, to turn the current of thought out of its smooth-worn track into untried and, at first, abrupt channels; and, the means were wanting. A total deficiency of all elementary books in our own language in which the modern improvements could be studied, precluded beginners from obtaining any glimpse beyond the narrow circle in

³² Waring, Edward, (1734-1798).

³³ Landen, John, (1719-1790).

which their teachers had revolved. The student is guided in his early choice of books by sanction and by usage. He may not, without hazard, venture to chalk out for himself a course of reading unusual and remote; and, rejecting the writers of his own country, choose foreigners for his instructors. To come to such a resolution presupposes a discrimination and a preference which is incompatible with entire unacquaintance with his subjects. It was only, therefore, when, although well instructed and perfect in the usual routine, he found himself arrested at the very first page of any of the elaborate works of the foreign geometers which chance might throw into his hands, that he could acquire the painful but necessary conviction of having all to begin afresh—much even to unlearn;—to forget habits—to change notations—to abandon points of view which had grown familiar—and, in short, put himself once more to school.

The late Professor Woodhouse seems to have been among the first of our countrymen who experienced this inward conviction, with its natural concomitant, the desire to propagate forward to other minds the rising impulse of our own. His papers on the independence of the analytical and geometrical modes of investigation, and the evidence of imaginary symbols, as well as his treatise on the principles of analytical calculation, contributed largely to produce this effect; and in his Trigonometry, in which, for the first time, this important part of geometry was placed before the English reader in a purely analytical form, and with all that peculiar grace and flexibility which belongs to it in that form, he conferred a most essential benefit on the elementary mathematics of his country. We owe also to him a treatise on the Calculus of Variations, not indeed very luminous, nor very extensive, but which had one pre-eminent merit, that of appearing at just the right moment, when the want of any work explanatory of what is merely technical in that calculus was becoming urgent.

An increasing interest in mathematical subjects was now also manifested by the occasional appearance of papers of a higher class in our learned Transactions, (such as that of Dr. Brinkley, now bishop of Cloyne, on the exponential developments of Lagrange, a memoir of curious and elaborate merit, and, though somewhat later in point of time, the curious investigations of Mr. Babbage on the theory of functional equations,) as well as of distinct works on subjects of pure analysis. The most remarkable of these is the *Essay on the various Orders of Logarithmic Transcendents*, by the late W. Spence of Greenock, the first formal essay in our language on any distinct and considerable branch of integral calculus, which had appeared since the publication of Hellins's papers on the *Rectification of the Conic Sections*. A premature death carried off, in Spence, one who might have become the ornament of his country in this department of knowledge. His posthumous essays, which were not, however, collected and published until 1819, prove him to have been both a learned and inventive analyst. He appears to have studied entirely without assistance, and to have formed his taste and strengthened his powers by a diligent perusal of the continental models. In consequence, he was enabled to attack questions which none of his countrymen had entered upon, such as the general integration of equations of finite differences, and others of that difficult and elevated class.

Among our Scottish countrymen, indeed, the torch of abstract science had never burnt so feebly nor decayed so far as in these southern abodes; nor was a high priest of the sublimer muse ever wanting in those ancient shrines, where Gregory and Napier had paid homage to her power. The late Professor Robinson, though his taste for the older geometry led him to undervalue both the evidence and the power of the modern analysis, was yet a mathematician of no inconsiderable note. The remarkable papers of Professor Playfair³⁴ on Porisms show how deeply

³⁴ See note 17, *Preliminary Dissertation*.

the mind of that sound mathematician and elegant writer was imbued with the spirit of the analytical methods and a sense of their superior power—a power, however, which he was content to admire and applaud, rather than ready to wield. It may indeed be questioned whether, by any researches of his own, however successful, he could have given a stronger impulse to the public mind in this direction than what his admirable review of the *Mécanique Céleste* communicated.

To this school also we owe the only British geometer who, at this period, seems to have possessed, not only a complete familiarity with the resources of the higher analysis, but also the habit of using them with skill and success in inquiries of moment in the system of the world.—we mean Professor Ivory. The appearance of his *Memoirs on the Attraction of Spheroids*, which are deservedly considered masterpieces of their kind, and which at once placed their author in the high rank among the geometers of Europe which he has ever since maintained, was almost simultaneous with that of Spence's work, a coincidence which might seem to warrant the most sanguine hopes of the speedy re-establishment of our mathematical glories. But the national taste and acquirements had sunk so low, that the stimulus of these examples was yet for a while unfelt. The *Essay on Logarithmic Transcendents* attracted little immediate notice, and the memoirs of Ivory, though received abroad with the respect and admiration they so justly merited, met with slender applause and no imitation at home. Their effect was, to seat their author on a solitary eminence, equally above the sympathy and the comprehension of the world around him. Since that period, however, a change has been slowly but steadily taking place in mathematical education. Students at our universities, fettered by no prejudices, entangled by no habits, and excited by the ardour and emulation of youth, had heard of the existence of masses of knowledge, from which they were debarred by the mere accident of position. There required no more. The *prestige* which magnifies what is unknown, and the attraction inherent in what is forbidden, coincided in their impulse. the books were procured and read, and produced their natural effects. The brows of many a Cambridge moderator were elevated, half in ire, half in admiration, at the unusual answers which began to appear in examination papers. Even moderators are not made of impenetrable stuff; their souls were touched, though fenced with sevenfold Jacquier, and tough bull-hide of Vince and Wood. they were carried away with the stream, in short, or replaced with successors full of their newly acquired powers. The modern analysis was adopted in its largest extent, and at this moment we believe that there exists not throughout Europe a centre from which a richer and purer light of mathematical instruction emanates through a community, than one, at least, of our universities.

One of the immediate consequences of the increased demand for a knowledge of the continental analysis, and the manner in which it is made subservient to physical inquiry, was a rapid and abundant supply of elementary works. Lacroix's lesser treatise (we wish it had been greater) has been translated, with note and comment, from the French, and Meier Hirsch's admirable work on the Theory of Algebraic Equations, from the German; and, in addition to these transplanted authorities, (the former of which may be regarded as having greatly contributed, by its numerous examples, to the final *domestication* of the peculiar notation of the differential calculus among us,) a host of indigenous ones on almost every branch of the pure and applied mathematics have emanated chiefly, but by no means entirely, from the press of the Cambridge University, which has thus signalled itself in a manner equally useful to the country and honourable to its directors. many of these works bear, it is true, strong and singular marks of the *transition state* of the science in which they were produced, but, on the whole they contain a copious body of instruction; and although we have still nothing approaching in extent and

excellence to the elementary works of Euler,³⁵ or to the superb digest of analytical knowledge contained in the great work of Lacroix,³⁶ to which we have alluded, yet, at least, our students can no longer complain of being left wholly without a guide, or without preparation for a profounder course of reading, should they feel disposed to enter upon it.

Another consequence, no less natural and obvious, of this altered state of feeling and instruction has been the gradual formation of what, at length, begins to merit the appellation of a British School of Geometry. We are far indeed from hoping soon to outstrip those who have so much the start of us, but the race is at least less hopeless than heretofore. The interval between the competitors has begun sensibly to diminish, and we need, at least, no longer fear being disgracefully distanced. We no longer perceive the same shyness, on the part of our mathematical champions, in entering on the great and vexed questions of the lunar and planetary perturbations, the theory of the tides, and others relating to the system of the world; nor the same indifference on that of the bystanders whether they are successful or no. The eminent geometer whom we have before named is no longer the only one among us who adventures himself fairly and boldly within the magic circle. On the contrary, we have recently witnessed the publication, by one of our countrymen, of several profound memoirs on the most intricate and profound parts of the terrestrial and planetary theory; on that of another, the novel, and, since Newton's time, the *unique* fact, of a new planetary inequality, not only detected, as so many have been, by British observation, but successfully referred to its origin, and subjected to exact calculation by British analysis, and that by no trifling effort or command of its resources.

We are very sure that in speaking so decidedly as we have felt compelled to do of the long-subsisting superiority of foreign mathematics to our own we run no hazard of wounding any feeling we wish to spare. Had our prospects, indeed, remained in the same deplorable state into which, but a very few years ago, they seem to have settled, we should perhaps preferred silence to the discouraging task of attempting to arouse an apathy so profound—but a better era is evidently advancing. The auguries are favourable. We hail them with delight, and we feel at the same time assured that our Airys, our Lubbocks,³⁷ our Hamiltons, and our Challises, the hope of our reviving geometry, will bear us out in the view we have taken, and acknowledge with gratitude and pleasure the sources whence they have drawn those principles they are now using so emulously and well.

Meanwhile the anomalous state of our mathematical literature which we have above described explains very naturally, what must have struck most mathematical readers as a remarkable feature in it, —we mean, the scanty supply of English works illustrative of the celestial mechanism, whether in the nature of express commentary and avowed illustration of the immortal work of Laplace, or in the form of independent treatises, calculated to bring the whole subject before the reader in a more compendious and explanatory manner than was compatible with Laplace's object, with the greatness and sweeping generality of his outline, or the close and laboured filling in of his detail. The *Elementary Illustrations of the Celestial Mechanics of Laplace*, by the late celebrated Dr. Young,³⁸ we hardly, we apprehend, be regarded by any reader as supplying satisfactorily the one of these desiderata; and although the Physical Astronomy of Professor Woodhouse approaches much nearer to what is requisite for the other, yet it by no means satisfies all, or nearly all, the conditions which such a work should accomplish. The

³⁵ See note 6, *Bk. I, Chap. II.*

³⁶ See note 26, *Bk. I, Chap. II.*

³⁷ See note 53, *Bk. II, Chap. VI.*

³⁸ See note 35, *Preliminary Dissertation.*

details of processes and developments into which it enters, though ample for elucidating the principles of the methods employed, is yet hardly sufficient to give a complete and effective grasp of the subject matter, while the combination of historical detail with theoretical elucidation, which it keeps in sight, tends to embarrass the reader by constantly shifting his point of view, and calling off his attention to inquire how mistakes have heretofore been committed and rectified; a most instructive thing in itself, no doubt—but calculated rather to render such a work a useful companion in a course of original reading, than to enable it to supply the place of many books, and offer, in a moderate compass, a compendium of what is known.

The works whose titles head the present article supply to the English reader, so far as they extend, both these desiderata, and supply them in a manner that leaves little to wish for. They are both, moreover, otherwise extremely remarkable in respect of the quarters from which they emanate. A lady, our own country woman, is the authoress of one; and to an American, by birth and residence, and to the American press, we stand indebted for the other. If anything were wanting to put our geometers effectively on their mettle, it would we think be found in such a coincidence.

Mrs. Somerville is already advantageously known to the philosophical world by her experiments on the magnetising influence of the violet rays of the solar spectrum; a delicate and difficult subject of physical inquiry, which the rarity of opportunities for its prosecution, arising from the nature of our climate, will allow no one to study in this country except at a manifest disadvantage. It is not surprising, therefore, that the feeble, although unequivocal indications of magnetism, which she undoubtedly obtained, should have been regarded by many as insufficient to decide the question at issue. To us their evidence appears entitled to considerable weight; but it is more to our immediate purpose to notice here, the simple and rational manner in which those experiments were conducted—the absence of needless complication and refinement in their plan, and of unnecessary or costly apparatus in their execution—and the perfect freedom from all pretension or affected embarrassment in their statement. The same simplicity of character and conduct, the same entire absence of anything like female vanity or affectation, pervades the whole of the present work. In the pursuit of her object, and in the natural and commendable wish to embody her acquired knowledge in an useful and instructive form for others, she seems entirely to have lost sight of herself; and, although in perfect consciousness of the possession of powers fully adequate to meet every exigency of her arduous undertaking, it yet never appears to have suggested itself to her mind, that the acquisition of such knowledge, or the possession of such powers, by a person of her sex, is in itself anything extraordinary or remarkable. We find, accordingly, beyond the name in the title page, nothing throughout the work introduced to remind us of its coming from a female hand. Even the tempting opportunity of deprecating criticism, which a preface affords, is neglected; nor does anything apologetic, in the tone of her admirably written preliminary discourse, betray a latent consciousness of superiority to the less-gifted of her sex, or a claim either on the admiration or forbearance of ours, beyond what the fair merits of the work itself may justly entitle it to. There is not only good taste, but excellent good sense in this. Whether admiration is due, or allowances needed, we accord both the one and the other, with perfect readiness, when left to the workings of our own good feeling. On the other hand, whenever we see such things as the poems of a minor, or the learning of a lady, introduced by an appeal, direct or indirect, to our good nature, we enter on our task of perusal with no very pleasant impression that this admirable weakness of our disposition is about to be largely taxed—an expectation in which, sooth to say, we are rarely disappointed.

In the present instance, however, we are neither called on for allowances, nor do we find any to make: on the contrary, we know not the geometer in this country who might not reasonably congratulate himself on the execution of such a work. The volume is dedicated to Lord Brougham, and appears to have been originally undertaken, at his instance, for publication by the Society for the Diffusion of Useful Knowledge; but the views of the author extending with its progress, it outgrew its first destination, and assumed an independent form. The nature of these views—the scope and object of the work—will perhaps be best understood from Mrs. Somerville’s own words:—

‘A complete acquaintance with Physical Astronomy can only be attained by those who are well versed in the higher branches of mathematical and mechanical science: such alone can appreciate the extreme beauty of the results, and of the means by which these results are obtained. Nevertheless a sufficient skill in analysis to follow the general outline, to see the mutual dependence of the different parts of the system, and to comprehend by what means some of the most extraordinary conclusions have been arrived at, is within the reach of many who shrink from the task, appalled by difficulties, which perhaps are not more formidable than those incident to the study of the elements of every branch of knowledge, and possibly overrating them by not making a sufficient distinction between the degree of mathematical acquirement necessary for making discoveries, and that which is requisite for understanding what others have done. That the study of mathematics and their application to astronomy are full of interest will be allowed by all who have devoted their time and attention to these pursuits, and they only can estimate the delight of arriving at truth, whether it be in the discovery of a world, or of a new property of numbers.’—p.2 (2nd edition).

Let us now see how far the conduct of Mrs. Somerville’s work corresponds with these views. In so doing, it is obvious that we are not to look for original discovery, the ambition of which is disclaimed, and which indeed would be misplaced in a work of the kind—nor even for absolute novelty in the methods of arriving at known results. The subject has been, in fact, so copiously handled, and by such a host of the most profound and accomplished mathematicians, that such novelty is now no longer to be expected, nor indeed desired in any fresh exposition of it. It is sufficient if all the results which it imports to know are clearly and perspicuously derived from their principles—the artifices of calculation on which their deduction rests, distinctly explained, and the processes actually pursued to such an extent as to give the reader a *thorough practical insight* into the development of the subject. This, we think, is fully accomplished in the work before us, for all those parts of the general subject which it professes to embrace, that is to say, the general exposition of the mechanical principles employed—the planetary and lunar theories, and those of Jupiter’s satellites with the incidental points arising naturally out of them. The development of the theory of the tides, and the precession of the equinoxes, the attraction of spheroids and the figure of the earth, appear to be reserved for a second volume. A certain degree of inconvenience is incurred by this in the investigation of those irregularities in the motions of the moon and satellites depending on the oblate form of their planets, which compels an anticipation of results not previously demonstrated; but this inconvenience is one more easily perceived than avoided.

In Mrs. Somerville’s preliminary dissertation, a general view is taken of the consequences of the law of gravitation, so far as they have hitherto been traced, whether as relates to the ecliptic motions and mutual perturbations of the planets and their satellites, and the

slow variations in the forms of their orbits thereby produced, or to the figures assumed by each of them individually, in consequence of the combination of their rotations on their axes with the attractions of their particles on each other and that of neighbouring bodies, together with the nutations, precessions, and librations of their axes themselves, arising from external actions, or, lastly, to the equilibrium and oscillations of the waters and atmospheres which cover their surfaces, comprehending the theory of the tides, and the great geological question of the general stability of the ocean. These, and the important points which are essentially dependent on such investigations—their application to those greater operations of geography to which the term geodesy is usually applied—to the determination of standards of weight and measure—to the fixation of chronological epochs—and a multitude of other interesting inquiries, are treated with a condensation, but at the same time a precision and clearness, which render this preliminary dissertation a model of its kind, and a most valuable acquisition to our literature. We have indeed no hesitation in saying, that we consider it by far the best condensed view of the Newtonian philosophy which has yet appeared. We do not, of course, mean to include the *Système du Monde* of Laplace himself, which embraces a far wider range, both of illustration and detail, and of which Mrs. Somerville's preface may in some sort be regarded as an abstract, but an abstract so vivid and judicious as to have all the merit of originality, and such as could have been produced only by one accustomed to large and general views, as well as perfectly familiar with the particulars of the subject.

As specimens of Mrs. Somerville's style of writing, we shall extract a few sentences almost from the commencement of this discourse:—

'Science, regarded as the pursuit of truth, which can only be attained by patient and unprejudiced investigation, wherein nothing is too great to be attempted, nothing so minute as to be justly disregarded, must ever afford occupation of consummate interest and of elevated meditation. The contemplation of the worlds of creation elevates the mind to the admiration of whatever is great and noble, accomplishing the object of all study, which in the elegant language of Sir James Mackintosh is to inspire the love of truth, of wisdom, of beauty, especially of goodness, the highest beauty, and of that supreme and eternal mind, which contains all truth and wisdom, all beauty and goodness. By the love or delightful contemplation and pursuit of these transcendent aims for their own sake only, the mind of man is raised from low and perishable objects, and prepared for those high destinies which are appointed for all those who are capable of them.'

We rejoice at this testimony to the intrinsic worth of scientific pursuits, and the pure and ennobling recompense they carry with them, from such a quarter. The female bosom is true to its impulses, and unwarping in their manifestation by motives which, in the sterner sex, are continually giving a bias to their estimates and conduct. The love of glory, the desire of practical utility, nay, even meaner and more selfish motives, may lead a man to toil in the pursuit of science, and adopt, without deeply feeling, the language of a disinterested worshipper at that sacred shrine—but we can conceive no motive, save immediate enjoyment of the kind so well described in the passage just quoted, which can induce a woman, especially an elegant and accomplished one, to undergo the severe and arduous mental exertion indispensable to the acquisition of a really profound knowledge of the higher analysis and its abstruser applications.

What follows is no less pleasing in another point of view:—

'The heavens afford the most sublime subject of study which can be derived from science: the magnitude and splendour of the objects, the inconceivable rapidity with which they move, and the enormous distances between them, impress the mind with some notion of the energy that maintains them in their motions with a durability to which we can see no limits. Equally conspicuous is the goodness of the great First Cause in having endowed man with faculties by which he can not only appreciate the magnificence of his works, but trace, with precision, the operation of his laws, use the globe he inhabits as a base wherewith to measure the magnitude and distance of the sun and planets, and make the diameter of the earth's orbit the first step of a scale by which he may ascend to the starry firmament. Such pursuits, while they ennoble the mind, at the same time inculcate humility, by showing that there is a barrier, which no energy, mental or physical, can ever enable us to pass: that however profoundly we may penetrate the depths of space, there still remain innumerable systems, compared with which those which seem so mighty to us must dwindle into insignificance, or even become invisible.'

We shall extract only one other passage from this discourse, as an example of the manner in which our fair authoress treats the less familiar topics, to which this part of her work is devoted. It is that in which the stability of the equilibrium of the seas and the permanence of the axis of the earth's rotation are considered.

'It appears from the marine shells found on the tops of the highest mountains, and in almost every part of the globe, that immense continents have been elevated above the ocean, which [ocean] ³⁹ must have engulfed others. Such a catastrophe would be occasioned by a variation in the position of the axis of rotation on the surface of the earth; for the seas tending to the new equator would leave some portions of the globe, and overwhelm others.

But theory proves that neither nutation, precession, nor any of the disturbing forces that affect the system, have the smallest influence on the axis of rotation, which maintains a permanent position on the surface, if the earth be not disturbed in its rotation by some foreign cause, as the collision of a comet which may have happened in the immensity of time. Then indeed, the equilibrium could only have been restored by the rushing of the seas to the new equator, which they would continue to do, till the surface was every where perpendicular to the direction of gravity. But it is probable that such an accumulation of the waters would not be sufficient to restore equilibrium if the derangement had been great; for the mean density of the sea is only about a fifth part of the mean density of the earth, and the mean depth even of the Pacific ocean is not more than four miles, whereas the equatorial radius of the earth exceeds the polar radius by twenty-five or thirty miles; consequently the influence of the sea on the direction of gravity is very small; and as it appears that a great change in the position of the axes is incompatible with the law of equilibrium, the geological phenomena must be ascribed to an internal cause. Thus amidst the mighty revolutions which have swept innumerable races of organized beings from the earth, which have elevated plains, and buried mountains in the ocean, the rotation of the earth, and the position of the axis on its surface, have undergone but slight variations.'

We will only pause to remark here, that an argument, which appears to us much more conclusive against the fact of any disturbance having, in remote antiquity, taken place in the axis of the earth's rotation, is to be found in the amount of the lunar irregularities which depend on

³⁹ Added by Herschel.

the earth's spheroidal figure. However insufficient the mere transfer of the mass of the ocean from the old to the new equator might be to ensure the permanence of the new axis, the enormous abrasion of the solid matter of such immensely-protuberant continents, as would, on that supposition, be left, by the violent and constant fluctuation of an unequilibrated ocean, would, (according to an ingenious remark of Professor Playfair,⁴⁰) no doubt, in the lapse of some ages, remodel the surface to the spheroidal form; but the lunar theory teaches us that the *internal strata*, as well as the *external outline*, of our globe, are elliptical, their centres being coincident and their axes identical with that of the surface,—a state of things incompatible with a subsequent accommodation of the surface to a new and different state of rotation from that which determined the original distribution of the component matter.

Mrs. Somerville's work is divided into four books, of which the first is devoted to the establishment of those general relations which prevail in the equilibrium or motion of bodies, or systems of bodies, whether solid or fluid, which are necessary to serve as a groundwork for the subsequent investigations;—the second, to the planetary theory, the elliptic motions and mutual perturbations of the bodies of our system, and the secular changes which take place in their orbits. The third book is given to the lunar theory; and the fourth to that of Jupiter's satellites, which is now for the first time introduced in any regular and extensive form to the English reader. From some confusion in the arrangement, or at least the numbering of the chapters in this book, it would seem to have been the original intention of the author to have thrown these two divisions of her subject into one, probably under the general head of the theory of Satellites. The actual arrangement is, on every account, infinitely preferable.

In the treatment of the statical and dynamical principles developed in the first part, the processes of the first book of the *Mécanique Céleste* are pretty closely but by no means servilely adhered to. Laplace's demonstration, for instance, of the fundamental principle of the composition of forces is suppressed, and its place supplied by one more elementary; and again, in the investigation of the equation of continuity of a fluid, the excessive difficulty and complication of the analysis by which he arrives at this result is evaded, and the whole subject in consequence greatly simplified by adopting a different and easier method of estimating the volume of an elementary molecule of the fluid in its displaced position. The whole of this portion of the work is also copiously illustrated by diagrams, which, however readily dispensed with by those with whom long habit has rendered familiar with analytical mechanics, are yet extremely useful in assisting the conception of less experienced readers. We could wish that a little more assistance of this kind had been afforded, and altogether a little more explanatory illustration bestowed on that chapter which treats of the rotatory motion of a solid mass. The subject needs it. There is a difficulty of conception in the notion of an axis of rotation shifting its position within a solid from instant to instant, as well as that of pressures exerted by the revolving matter, on such an imaginary and fugitive line, which is very embarrassing to one not accustomed to such speculations, though easily removed by dilating a little on the subject, and placing it in different and familiar points of view. We have always considered this part of analytical mechanics as among the most beautiful and exquisite of its applications. It is usually, however, regarded by beginners as more abstruse than its real difficulties authorize. This arises partly from the obscurity of conception we have alluded to, but partly, also, from a more technical cause,—the frequent changes of co-ordinates which its analytical treatment involves. This is a difficulty of the same kind as transposition, in a musical performance, from one key to another; and as a

⁴⁰ See note 17, *Preliminary Dissertation*.

musician can never expect to become a ready performer till practice has made such difficulty vanish, so the mathematical student can never feel at complete ease in the higher applications, till all such mere technical evolutions cease to be complained of as difficulties, or even felt as inconveniences.

We could have wished, too, that instead of entering, in this part of the work, on the theory of tides, which is by far the most complicated and infinitely the least satisfactory part of the general subject, that of the attractions of spheroids had been traced, at least so far as to demonstrate the theorems which are afterwards taken for granted in the development of those terms of the mercurial and lunar theory, and that of Jupiter's satellites, which depend on the oblate figure of the primary. As it is only a single term in the development of the series expressing the deviation of the law of gravity in the spheroid from that in the sphere which is wanted, this might have been very easily done, and at the same time the reader prepared to enter more fully into this interesting part of the subject, in a more advanced state of his knowledge.

In the second book the planetary theory is given with a fullness commensurate with its importance. Its first chapters are of course devoted to the theory of elliptic motion, which is concisely, but very perspicuously stated. The equations used are the beautiful integrals of the general differential equations first obtained, if we remember rightly, by Lagrange, and used by him with such wonderful effect for ascertaining the variations of the elements. They are the same which Laplace derives in the 18th article of his second book, by a process which we should be inclined to tax with excessive and useless generality, were it not quite necessary to show that this important part of the theory had been probed to the quick, and every resource which analysis could furnish exhausted on it. Mrs. Somerville, however, very properly derives them by the ordinary processes of direct integration. The usual properties of elliptic motion, with the series for the developments of the anomalies and radius vector afterwards required, are there demonstrated, and a few pages added on the determination of the elements.

We should have been glad to have found in this part of the work some outline of the powerful and elegant researches of Gauss on the determination of the orbits of the celestial bodies, and especially some more practical method of determining those of comets than Laplace's. The subject of the motion of comets is, however, summarily dismissed; and even the beautiful theorem of Lambert,⁴¹ which expresses the time of describing a parabolic arc in terms of the radii vectores of its extremities and its chord, is omitted.

The fine idea of Lagrange, by which the perturbations of a planet are expressed by means of a variable ellipse, and all its inequalities referred to changes in the elliptic elements which are supposed to be in a state of continual fluctuation, has introduced a degree of simplicity and symmetry into the analytical treatment of the planetary theory such as could hardly have been hoped for, and divested it of all that was repulsive and much that was merely laborious in its investigation. It is in this view of the subject alone, that a neat conception can be formed of the distinction between variations truly secular, and those inequalities of long periods which were originally confounded with secular changes. The former class are those which are independent of the mutual configurations of the planets one among the other, and in their theory no other quantities enter than the elements themselves and the time; all those variables on which depend the situations of the planets in their orbits, such as their longitudes, latitudes, and distances from the sun, being excluded. The reactions contemplated in this part of the theory are not so much those of planet on planet, as of orbit on orbit. Nothing can be more exquisite in analysis, nothing more refined in conception, than this investigation, on which depend all those grand propositions

⁴¹ See note 2, *Bk. II, Chap. X.*

respecting the stability of the system to which we have already alluded. In the conduct of this part of her subject, Mrs. Somerville has chiefly adhered to the analysis of Lagrange, as stated by Laplace in the supplement to the third volume of the *Mécanique Céleste*, only in that important and difficult part of it which concerns the invariability of the axes as affected by the squares and products of the disturbing forces, availing herself of the subsequent elaborate investigations of Poisson.

The periodical part of the perturbations of the elements is next investigated, not so much with a view to the ultimate derivation of formulae for the practical computations of the longitudes and latitudes of the disturbed planets, which, though practicable, is not so easy in this view of the subject as in that of Laplace, which depends on the principle of successive approximations from the differential equations of the troubled orbit; and, so to speak, consists in a continual gathering up of the loose and raveled ends of the skein which appear in the form of unperiodic terms out of their proper place. The chief advantage of Lagrange's view of the subject when applied to the periodical terms, consists in the clear insight which it gives us into the nature of those equations of long period, such as, for instance, the secular equations, as they were formerly called, of Jupiter and Saturn, and the secular acceleration of the moon, which appear to alter the mean motion, and therefore to affect the axes of their orbits. They, in fact, do so; but such alterations are all periodical, and no way interfere with the general truth of their ultimate and average invariability. It ought to be remarked, however, that in the case of highly eccentric orbits, such as those of comets, which may approach very near the greater bodies of our system, deviations from the mean motion, and fluctuations of the periodic time may go to such an extent, and the compensation may be put off so long, that, although theoretically true, the proposition of the permanence of the axis may cease to have any useful or practical meaning. This is remarkably exemplified in the comet of Halley,⁴² whose periodic return is affected by inequalities of a great many months, nay even whole years.

In the actual development of the perturbation of a planet in longitude there is a term introduced, at the very first step, proportional to the time. This is, in fact, the representative of that part of the planetary action which, like the mean effect of the ablatitious force in Newton's lunar theory, tends to diminish or increase the average intensity of gravitation to the central body, and thereby alter the mean motion and period from what they would be had the disturbing body no existence. The nature of this term, which appears very obscure as it is disposed of in the *Mécanique Céleste*, is placed by Mrs. Somerville in a much clearer light—(p. 381. 2nd edition)

The developments of the perturbations in longitude, latitude, and distance, though tedious, intricate, and laborious, offer no points of real difficulty, except—, in respect of the terms proportional to powers of the time introduced by integration, for the treatment of which we are referred to Laplace's memoir, in which this difficulty was first obviated; secondly, in respect of terms which, from the near commensurability of the mean motions, acquire small divisors by integration. These are, of all which occur in the planetary theory, the most troublesome. In the case of Jupiter and Saturn they give rise to the 'great equation' of those planets, to which Mrs. Somerville has devoted a masterly chapter, where it is treated with much clearness, and in a very compact and well digested form. On the whole, we consider the development of the planetary theory, as we have it thus brought before us, to be extremely well performed, and, in fact, a most useful and valuable summary of the subject.

⁴² See Art. 618, Bk. II.

The lunar theory differs in many essential points from the planetary. This is owing to the rapid motion of the apsides and nodes of its orbit, in consequence of which it is impossible to treat it, as we do those of the planets, as an ellipse, subject to small and slow variations: this necessitates a totally different analytical treatment of the problem. That which has been universally followed since its first employment by d'Alembert, consists in expressing, not as for the planets, the longitude, &c. in functions of the time, but *vice versâ*, making the moon's longitude itself the independent variable, and expressing the time and the other co-ordinates in terms of this. The reversion of the first series, and substitution of the result in others, will then enable us to express all the co-ordinates in functions of the time.

Nothing, however, can be well imagined more formidable than the actual execution of these operations; at the same time that, when the delicacies of the management of the coefficients depending on the motions of the apsides and nodes are once understood, the whole is little more than a mechanical process, demanding only unwearied patience for its accomplishment. In the treatment, therefore, of this part of the subject, an author, whose object is merely to exhibit a clear view of processes, and a summary of results, is limited to a narrow path, affording little scope for the exercise of any facility but judgement in deciding where to stop. Mrs. Somerville seems to have considered it her duty here to err on the safe side; so that the equations of her lunar theory are, in fact, little else than a transcript, *mutatis mutandis*, of those of Laplace, and co-extensive with his formulae. She has, however, had recourse to the gigantic work of Damoiseau⁴³ for the expression of the longitude in terms of the time, the deduction of which, by the actual reversion of Laplace's series, would have been a work of infinitely too much labour, and which every one but those who make it their especial project to surpass all who have gone before them in this most intricate inquiry, must be content to receive on his authority.

The last division of the work is devoted to the theory of Jupiter's satellites—a curious and elegant system, in which the near approach to commensurability in the mean motions of the three interior satellites gives rise to peculiarities of a very remarkable nature both in the analysis and its results. In this system also the great ellipticity of the central body causes a material deviation in its attraction from the law of gravity, the effect being to introduce a term in the expression of the *perturbative function*, varying inversely as the cube of the distance. As we have before observed, the investigation of this term is not given, and we must, moreover, take this opportunity to notice that, by an inaccuracy of wording, which is repeated wherever the same point is referred to in other parts of the work, this term is always spoken of as expressing 'the attraction of the excess of matter at the equator of the central body,' whereas, in fact, it expresses no *attractive force* at all, but an artificial quantity, being the significant perturbative term in the development of that useful function in the theory of the attraction of spheroids, which expresses the sum of the molecules of the attracting body, divided each by its respective distance from the point attracted, and which is constantly employed by Laplace in this theory, in preference to the direct expression of the attraction itself, for the convenience and symmetry of analysis. We are the more particular in noticing this point, as the most considerable fault we have to find with the work before us consists in an habitual laxity of language, evidently originating in so complete a familiarity with the *quantities* concerned, as to induce a disregard of the *words* by which they are designated, but which, to any one less intimately conversant with the actual analytical operations than its author, must have infallibly become a source of serious errors, and which at all events, renders it necessary for the reader to be constantly on his guard. It would not be difficult to support this charge (which is rather a grave one) by citations, but we should be extremely

⁴³ See note 6, *Bk. III, Chap. I.*

unwilling to leave, at the conclusion of our article, any impression less agreeable than that of the unfeigned delight, and we may add, astonishment, with which the perusal of the work has filled us.

We must not, however, stop without saying something of Mr. Bowditch's⁴⁴ performance; though what we do say must be short. The idea of undertaking a translation of the whole *Mécanique Céleste*, accompanied throughout with a copious running commentary, is one which savours, at first sight, of the *gigantesque*, and is certainly one which, from what we had hitherto had reason to conceive of the popularity and diffusion of mathematical knowledge on the opposite shores of the Atlantic, we should never have expected to have found originated—or, at least, carried into execution, in that quarter. The first volume only has as yet reached us; and when we consider the great difficulty of printing works of this nature, to say nothing of the heavy and probably unremunerated expense, we are not surprised at the delay of the second. Meanwhile the part actually completed (which contains the first two books of Laplace's work) is, with few and slight exceptions, just what we could have wished to see—an exact and careful translation into very good English—exceedingly well printed, and accompanied with notes appended to each page, which leave no step in the text of moment unsupplied, and hardly any material difficulty either of conception or reasoning unelucidated. To the student of 'Celestial Mechanism' such a work must be invaluable, and we sincerely hope that the success of this volume, which seems thrown out to try the feeling of the public, both American and British, will be such as to induce the speedy appearance of the sequel. Should this unfortunately not be the case, we shall deeply lament that the liberal offer of the American Academy of Arts and Sciences, to print the whole at their expense, was not accepted. Be that as it may, it is impossible to regard the appearance of such a work, even in its present incomplete state, as otherwise than highly creditable to American science, and as the harbinger of future achievements in the loftiest fields of intellectual prowess. Here, at least, is an arena on which we may contend with an emulation unembittered by rivalry.—'Whatever,' says Delambre, 'be the state of political relations, the sciences ought to form, among those who cultivate them, a republic essentially at peace within itself,'—a sentiment applicable, doubtless, to all, but pre-eminently so to that calm, dispassionate pursuit of truth which forms the very essence of the abstract sciences.

⁴⁴ See note 3, *Foreword to the Second Edition*.